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FASTCoordinateSystems.doc
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The following are transformation equations defining the angular orientation of each coordinate system inherent in FAST.

Before providing these, it is useful to discuss the transformation equation relating coordinate system $\boldsymbol{x}$ to coordinate system $\boldsymbol{X}$ where $\boldsymbol{x}$ (with orthogonal axes $\boldsymbol{x}_1$, $\boldsymbol{x}_2$, and $\boldsymbol{x}_3$) is the coordinate system resulting from three rotations (θ_1, θ_2, and θ_3) about the orthogonal axes ($\boldsymbol{X}_1$, $\boldsymbol{X}_2$, and $\boldsymbol{X}_3$) of coordinate system $\boldsymbol{X}$. With all rotation angles assumed to be small, the order of rotations does not matter and Euler angles do not need to be used. Instead, what we want, is a transformation equation that is consistent with classical Bernoulli-Euler beam theory (which assumes small rotations). The correct transformation equation is:

$$\begin{Bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \boldsymbol{x}_3 \end{Bmatrix} \approx \underbrace{\begin{bmatrix} 1 & \theta_3 & -\theta_2 \\ -\theta_3 & 1 & \theta_1 \\ \theta_2 & -\theta_1 & 1 \end{bmatrix}}_{[A]} \begin{Bmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \\ \boldsymbol{X}_3 \end{Bmatrix},$$

where $[A]$ is referred to as the Bernoulli-Euler transformation matrix in this work. The approximation symbol $(\approx)$ is used in place of an equals symbol $(=)$ in the above expression since $[A]$ is not orthonormal, which implies that the resulting $\boldsymbol{x}$ from this expression is not made up of a set of mutually orthogonal axes (all transformation matrices between sets of mutually orthogonal axes must be orthonormal). So it is evident that in place of $[A]$, what we want is the closest orthonormal matrix to $[A]$, which is referred to as $[TransMat]$ in this work. From linear algebra, we know that the closest orthonormal matrix to $[A]$ in the Frobenius Norm sense is:

$$[TransMat] = [U][V]^T,$$

where the columns of $[U]$ contain the eigenvectors of $[A][A]^T$ and the columns of $[V]$ contain the eigenvectors of $[A]^T[A]$. This result follows directly from the Singular Value Decomposition (SVD) of $[A]$:

$$[A] = [U][\Sigma][V]^T,$$

where $[\Sigma]$ is a diagonal matrix containing the singular values of $[A]$, which are $\sqrt{eigenvalues\ of\ [A][A]^T} = \sqrt{eigenvalues\ of\ [A]^T[A]}$.

The algebraic form of the resulting transformation matrix is:

$$\begin{Bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \boldsymbol{x_3} \end{Bmatrix} = \underbrace{\begin{bmatrix} \frac{\theta_1^2 \sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}+\theta_2^2+\theta_3^2}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} & \frac{\theta_3\left(\theta_1^2+\theta_2^2+\theta_3^2\right)+\theta_1\theta_2\left(\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}-1\right)}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} & \frac{-\theta_2\left(\theta_1^2+\theta_2^2+\theta_3^2\right)+\theta_1\theta_3\left(\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}-1\right)}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} \\ \frac{-\theta_3\left(\theta_1^2+\theta_2^2+\theta_3^2\right)+\theta_1\theta_2\left(\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}-1\right)}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} & \frac{\theta_1^2+\theta_2^2\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}+\theta_3^2}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} & \frac{\theta_1\left(\theta_1^2+\theta_2^2+\theta_3^2\right)+\theta_2\theta_3\left(\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}-1\right)}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} \\ \frac{\theta_2\left(\theta_1^2+\theta_2^2+\theta_3^2\right)+\theta_1\theta_3\left(\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}-1\right)}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} & \frac{-\theta_1\left(\theta_1^2+\theta_2^2+\theta_3^2\right)+\theta_2\theta_3\left(\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}-1\right)}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} & \frac{\theta_1^2+\theta_2^2+\theta_3^2\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} \end{bmatrix}}_{[TransMat]} \begin{Bmatrix} \boldsymbol{X_1} \\ \boldsymbol{X_2} \\ \boldsymbol{X_3} \end{Bmatrix}$$

This was derived symbolically by J. Jonkman by computing $[U][V]^T$ by hand with verification in Mathematica.

Tower Base / Platform Coordinate System

$$\begin{Bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \\ \boldsymbol{a_3} \end{Bmatrix} = \left[TransMat\left(\theta_1 = q_R, \theta_2 = q_Y, \theta_3 = -q_P\right)\right] \begin{Bmatrix} \boldsymbol{z_1} \\ \boldsymbol{z_2} \\ \boldsymbol{z_3} \end{Bmatrix}$$

Tower Element-Fixed Coordinate System

$$\begin{Bmatrix} \boldsymbol{t_1}(h) \\ \boldsymbol{t_2}(h) \\ \boldsymbol{t_3}(h) \end{Bmatrix} = \left[TransMat\left(\theta_1 = \theta_{SS}(h), \theta_2 = 0, \theta_3 = \theta_{FA}(h)\right)\right] \begin{Bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \\ \boldsymbol{a_3} \end{Bmatrix}$$

where,

$\theta_{FA}(h) = -\left[\frac{d\phi_1^{TFA}(h)}{dh} q_{TFA1} + \frac{d\phi_2^{TFA}(h)}{dh} q_{TFA2} \right]$ and $\theta_{SS}(h) = \left[\frac{d\phi_1^{TSS}(h)}{dh} q_{TSS1} + \frac{d\phi_2^{TSS}(h)}{dh} q_{TSS2} \right]$

Tower-Top / Base Plate Coordinate System

$$\begin{Bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \boldsymbol{b}_3 \end{Bmatrix} = \left[TransMat\left(\theta_1 = \theta_{SS}\left(TwrFlexL\right), \theta_2 = 0, \theta_3 = \theta_{FA}\left(TwrFlexL\right)\right) \right] \begin{Bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{a}_3 \end{Bmatrix}$$

where,

$$\theta_{FA}\left(TwrFlexL\right) = -\left[\left. \frac{d\phi_1^{TFA}(h)}{dh} \right|_{h=TwrFlexL} q_{TFA1} + \left. \frac{d\phi_2^{TFA}(h)}{dh} \right|_{h=TwrFlexL} q_{TFA2} \right] \text{ and } \theta_{SS}\left(TwrFlexL\right) = \left[\left. \frac{d\phi_1^{TSS}(h)}{dh} \right|_{h=TwrFlexL} q_{TSS1} + \left. \frac{d\phi_2^{TSS}(h)}{dh} \right|_{h=TwrFlexL} q_{TSS2} \right]$$

Nacelle / Yaw Coordinate System

$$\begin{Bmatrix} \boldsymbol{d}_1 \\ \boldsymbol{d}_2 \\ \boldsymbol{d}_3 \end{Bmatrix} = \begin{bmatrix} \cos\left(q_{Yaw}\right) & 0 & -\sin\left(q_{Yaw}\right) \\ 0 & 1 & 0 \\ \sin\left(q_{Yaw}\right) & 0 & \cos\left(q_{Yaw}\right) \end{bmatrix} \begin{Bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \boldsymbol{b}_3 \end{Bmatrix}$$

Rotor-Furl Coordinate System

$$\begin{Bmatrix} \boldsymbol{rf}_1 \\ \boldsymbol{rf}_2 \\ \boldsymbol{rf}_3 \end{Bmatrix} = \begin{bmatrix} \begin{matrix} \left[1-\cos^2\left(RFrlSkew\right)\cos^2\left(RFrlTilt\right)\right]\cos\left(q_{RFrl}\right) \\ +\cos^2\left(RFrlSkew\right)\cos^2\left(RFrlTilt\right) \end{matrix} & \begin{matrix} \cos\left(RFrlSkew\right)\cos\left(RFrlTilt\right)\sin\left(RFrlTilt\right)\left[1-\cos\left(q_{RFrl}\right)\right] \\ -\sin\left(RFrlSkew\right)\cos\left(RFrlTilt\right)\sin\left(q_{RFrl}\right) \end{matrix} & \begin{matrix} \cos\left(RFrlSkew\right)\sin\left(RFrlSkew\right)\cos^2\left(RFrlTilt\right)\left[\cos\left(q_{RFrl}\right)-1\right] \\ -\sin\left(RFrlTilt\right)\sin\left(q_{RFrl}\right) \end{matrix} \\ \begin{matrix} \cos\left(RFrlSkew\right)\cos\left(RFrlTilt\right)\sin\left(RFrlTilt\right)\left[1-\cos\left(q_{RFrl}\right)\right] \\ +\sin\left(RFrlSkew\right)\cos\left(RFrlTilt\right)\sin\left(q_{RFrl}\right) \end{matrix} & \cos^2\left(RFrlTilt\right)\cos\left(q_{RFrl}\right)+\sin^2\left(RFrlTilt\right) & \begin{matrix} \sin\left(RFrlSkew\right)\cos\left(RFrlTilt\right)\sin\left(RFrlTilt\right)\left[\cos\left(q_{RFrl}\right)-1\right] \\ +\cos\left(RFrlSkew\right)\cos\left(RFrlTilt\right)\sin\left(q_{RFrl}\right) \end{matrix} \\ \begin{matrix} \cos\left(RFrlSkew\right)\sin\left(RFrlSkew\right)\cos^2\left(RFrlTilt\right)\left[\cos\left(q_{RFrl}\right)-1\right] \\ +\sin\left(RFrlTilt\right)\sin\left(q_{RFrl}\right) \end{matrix} & \begin{matrix} \sin\left(RFrlSkew\right)\cos\left(RFrlTilt\right)\sin\left(RFrlTilt\right)\left[\cos\left(q_{RFrl}\right)-1\right] \\ -\cos\left(RFrlSkew\right)\cos\left(RFrlTilt\right)\sin\left(q_{RFrl}\right) \end{matrix} & \begin{matrix} \left[1-\sin^2\left(RFrlSkew\right)\cos^2\left(RFrlTilt\right)\right]\cos\left(q_{RFrl}\right) \\ +\sin^2\left(RFrlSkew\right)\cos^2\left(RFrlTilt\right) \end{matrix} \end{bmatrix} \begin{Bmatrix} \boldsymbol{d}_1 \\ \boldsymbol{d}_2 \\ \boldsymbol{d}_3 \end{Bmatrix}$$

Shaft Coordinate System

$$\begin{Bmatrix} \boldsymbol{c}_1 \\ \boldsymbol{c}_2 \\ \boldsymbol{c}_3 \end{Bmatrix} = \begin{bmatrix} \cos\left(ShftSkew\right)\cos\left(ShftTilt\right) & \sin\left(ShftTilt\right) & -\sin\left(ShftSkew\right)\cos\left(ShftTilt\right) \\ -\cos\left(ShftSkew\right)\sin\left(ShftTilt\right) & \cos\left(ShftTilt\right) & \sin\left(ShftSkew\right)\sin\left(ShftTilt\right) \\ \sin\left(ShftSkew\right) & 0 & \cos\left(ShftSkew\right) \end{bmatrix} \begin{Bmatrix} \boldsymbol{rf}_1 \\ \boldsymbol{rf}_2 \\ \boldsymbol{rf}_3 \end{Bmatrix}$$

Azimuth Coordinate System

$$\begin{Bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\left(q_{DrTr}+q_{GeAz}\right) & \sin\left(q_{DrTr}+q_{GeAz}\right) \\ 0 & -\sin\left(q_{DrTr}+q_{GeAz}\right) & \cos\left(q_{DrTr}+q_{GeAz}\right) \end{bmatrix} \begin{Bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \end{Bmatrix}$$

Teeter Coordinate System

$$\begin{Bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \end{Bmatrix} = \begin{bmatrix} \cos\left(q_{Teet}\right) & 0 & -\sin\left(q_{Teet}\right) \\ 0 & 1 & 0 \\ \sin\left(q_{Teet}\right) & 0 & \cos\left(q_{Teet}\right) \end{bmatrix} \begin{Bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{Bmatrix}$$

Hub / Delta-3 Coordinate System

$$\begin{Bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \mathbf{g}_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\left(Delta3\right) & \sin\left(Delta3\right) \\ 0 & -\sin\left(Delta3\right) & \cos\left(Delta3\right) \end{bmatrix} \begin{Bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \end{Bmatrix}$$

Hub (Prime) Coordinate System

$$\begin{Bmatrix} \mathbf{g'}_1^{B1} \\ \mathbf{g'}_2^{B1} \\ \mathbf{g'}_3^{B1} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \mathbf{g}_3 \end{Bmatrix}$$

The equation for $\mathbf{g'}^{B2}$ of blade 2 is similar.

Coned Coordinate System

$$\begin{Bmatrix} i_1^{B1} \\ i_2^{B1} \\ i_3^{B1} \end{Bmatrix} = \begin{bmatrix} \cos\left[PreCone(1)\right] & 0 & -\sin\left[PreCone(1)\right] \\ 0 & 1 & 0 \\ \sin\left[PreCone(1)\right] & 0 & \cos\left[PreCone(1)\right] \end{bmatrix} \begin{Bmatrix} g'^{B1}_1 \\ g'^{B1}_2 \\ g'^{B1}_3 \end{Bmatrix}$$

The equation for $\boldsymbol{i}^{B2}$ is similar.

Blade / Pitched Coordinate System

$$\begin{Bmatrix} j_1^{B1} \\ j_2^{B1} \\ j_3^{B1} \end{Bmatrix} = \begin{bmatrix} \cos\left[BlPitch(1)\right] & -\sin\left[BlPitch(1)\right] & 0 \\ \sin\left[BlPitch(1)\right] & \cos\left[BlPitch(1)\right] & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} i_1^{B1} \\ i_2^{B1} \\ i_3^{B1} \end{Bmatrix}$$

The equation for $\boldsymbol{j}^{B2}$ is similar.

Blade Coordinate System Aligned with Local Structural Axes (not element fixed)

$$\begin{Bmatrix} Lj_1^{B1}(r) \\ Lj_2^{B1}(r) \\ Lj_3^{B1}(r) \end{Bmatrix} = \begin{bmatrix} \cos\left[\theta_S^{B1}(r)\right] & -\sin\left[\theta_S^{B1}(r)\right] & 0 \\ \sin\left[\theta_S^{B1}(r)\right] & \cos\left[\theta_S^{B1}(r)\right] & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} i_1^{B1} \\ i_2^{B1} \\ i_3^{B1} \end{Bmatrix}$$

The equation for $\boldsymbol{Lj}^{B2}(r)$ is similar.

Blade Element-Fixed Coordinate System Aligned with Local Structural Axes

$$\begin{Bmatrix} n_1^{B1}(r) \\ n_2^{B1}(r) \\ n_3^{B1}(r) \end{Bmatrix} = \left[TransMat\left(\theta_1 = \theta_x^{B1}(r), \theta_2 = \theta_y^{B1}(r), \theta_3 = 0\right)\right] \begin{Bmatrix} Lj_1^{B1}(r) \\ Lj_2^{B1}(r) \\ Lj_3^{B1}(r) \end{Bmatrix}$$

where,

$$\theta_x^{B1}(r)=cos\left[\theta_S^{B1}(r)\right]\theta_{IP}^{B1}(r)-sin\left[\theta_S^{B1}(r)\right]\theta_{OoP}^{B1}(r),\quad \theta_y^{B1}(r)=sin\left[\theta_S^{B1}(r)\right]\theta_{IP}^{B1}(r)+cos\left[\theta_S^{B1}(r)\right]\theta_{OoP}^{B1}(r),\ \text{and}$$

$$\theta_{IP}^{B1}(r)=-\left[\frac{d\psi_1^{B1}(r)}{dr}q_{B1F1}+\frac{d\psi_2^{B1}(r)}{dr}q_{B1F2}+\frac{d\psi_3^{B1}(r)}{dr}q_{B1E1}\right],\quad \theta_{OoP}^{B1}(r)=\left[\frac{d\phi_1^{B1}(r)}{dr}q_{B1F1}+\frac{d\phi_2^{B1}(r)}{dr}q_{B1F2}+\frac{d\phi_3^{B1}(r)}{dr}q_{B1E1}\right]$$

The equation for $\boldsymbol{n}^{B2}(r)$ is similar.

<u>Blade Element-Fixed Coordinate System Used for Calculating and Returning Aerodynamic Loads</u>

This coordinate system is coincident with $\boldsymbol{i}^{B1}$ when the blade is undeflected.

$$\begin{Bmatrix}\boldsymbol{m}_1^{B1}(r)\\ \boldsymbol{m}_2^{B1}(r)\\ \boldsymbol{m}_3^{B1}(r)\end{Bmatrix}=\begin{bmatrix}cos\left[BlPitch(1)+\theta_S^{B1}(r)\right] & sin\left[BlPitch(1)+\theta_S^{B1}(r)\right] & 0\\ -sin\left[BlPitch(1)+\theta_S^{B1}(r)\right] & cos\left[BlPitch(1)+\theta_S^{B1}(r)\right] & 0\\ 0 & 0 & 1\end{bmatrix}\begin{Bmatrix}\boldsymbol{n}_1^{B1}(r)\\ \boldsymbol{n}_2^{B1}(r)\\ \boldsymbol{n}_3^{B1}(r)\end{Bmatrix}$$

The equation for $\boldsymbol{m}^{B2}(r)$ is similar.

<u>Blade Element-Fixed Coordinate System Aligned with Local Aerodynamic Axes (i.e., chordline) / Trailing Edge Coordinate System</u>

$$\begin{Bmatrix}\boldsymbol{te}_1^{B1}(r)\\ \boldsymbol{te}_2^{B1}(r)\\ \boldsymbol{te}_3^{B1}(r)\end{Bmatrix}=\begin{bmatrix}cos\left[BlPitch(1)+\theta_A^{B1}(r)\right] & -sin\left[BlPitch(1)+\theta_A^{B1}(r)\right] & 0\\ sin\left[BlPitch(1)+\theta_A^{B1}(r)\right] & cos\left[BlPitch(1)+\theta_A^{B1}(r)\right] & 0\\ 0 & 0 & 1\end{bmatrix}\begin{Bmatrix}\boldsymbol{m}_1^{B1}(r)\\ \boldsymbol{m}_2^{B1}(r)\\ \boldsymbol{m}_3^{B1}(r)\end{Bmatrix}$$

The equation for $\boldsymbol{te}^{B2}(r)$ is similar.

Tail-Furl Coordinate System

$$\begin{Bmatrix} \boldsymbol{tf_1} \\ \boldsymbol{tf_2} \\ \boldsymbol{tf_3} \end{Bmatrix} = \begin{bmatrix} \begin{matrix} \left[1-\cos^2(TFrlSkew)\cos^2(TFrlTilt)\right]\cos(q_{TFrl}) \\ +\cos^2(TFrlSkew)\cos^2(TFrlTilt) \end{matrix} & \begin{matrix} \cos(TFrlSkew)\cos(TFrlTilt)\sin(TFrlTilt)\left[1-\cos(q_{TFrl})\right] \\ -\sin(TFrlSkew)\cos(TFrlTilt)\sin(q_{TFrl}) \end{matrix} & \begin{matrix} \cos(TFrlSkew)\sin(TFrlSkew)\cos^2(TFrlTilt)\left[\cos(q_{TFrl})-1\right] \\ -\sin(TFrlTilt)\sin(q_{TFrl}) \end{matrix} \\ \begin{matrix} \cos(TFrlSkew)\cos(TFrlTilt)\sin(TFrlTilt)\left[1-\cos(q_{TFrl})\right] \\ +\sin(TFrlSkew)\cos(TFrlTilt)\sin(q_{TFrl}) \end{matrix} & \cos^2(TFrlTilt)\cos(q_{TFrl})+\sin^2(TFrlTilt) & \begin{matrix} \sin(TFrlSkew)\cos(TFrlTilt)\sin(TFrlTilt)\left[\cos(q_{TFrl})-1\right] \\ +\cos(TFrlSkew)\cos(TFrlTilt)\sin(q_{TFrl}) \end{matrix} \\ \begin{matrix} \cos(TFrlSkew)\sin(TFrlSkew)\cos^2(TFrlTilt)\left[\cos(q_{TFrl})-1\right] \\ +\sin(TFrlTilt)\sin(q_{TFrl}) \end{matrix} & \begin{matrix} \sin(TFrlSkew)\cos(TFrlTilt)\sin(TFrlTilt)\left[\cos(q_{TFrl})-1\right] \\ -\cos(TFrlSkew)\cos(TFrlTilt)\sin(q_{TFrl}) \end{matrix} & \begin{matrix} \left[1-\sin^2(TFrlSkew)\cos^2(TFrlTilt)\right]\cos(q_{TFrl}) \\ +\sin^2(TFrlSkew)\cos^2(TFrlTilt) \end{matrix} \end{bmatrix} \begin{Bmatrix} \boldsymbol{d_1} \\ \boldsymbol{d_2} \\ \boldsymbol{d_3} \end{Bmatrix}$$

Tail Fin Coordinate System

$$\begin{Bmatrix} \boldsymbol{p_1} \\ \boldsymbol{p_2} \\ \boldsymbol{p_3} \end{Bmatrix} = \begin{bmatrix} \cos(TFinSkew)\cos(TFinTilt) & \sin(TFinTilt) & -\sin(TFinSkew)\cos(TFinTilt) \\ \begin{matrix} \sin(TFinSkew)\sin(TFinBank) \\ -\cos(TFinSkew)\sin(TFintTilt)\cos(TFinBank) \end{matrix} & \cos(TFinTilt)\cos(TFinBank) & \begin{matrix} \cos(TFinSkew)\sin(TFinBank) \\ +\sin(TFintSkew)\sin(TFinTilt)\cos(TFinBank) \end{matrix} \\ \begin{matrix} \sin(TFinSkew)\cos(TFinBank) \\ +\cos(TFinSkew)\sin(TFintTilt)\sin(TFinBank) \end{matrix} & -\cos(TFinTilt)\sin(TFinBank) & \begin{matrix} \cos(TFinSkew)\cos(TFinBank) \\ -\sin(TFintSkew)\sin(TFinTilt)\sin(TFinBank) \end{matrix} \end{bmatrix} \begin{Bmatrix} \boldsymbol{tf_1} \\ \boldsymbol{tf_2} \\ \boldsymbol{tf_3} \end{Bmatrix}$$