Jason Jonkman
FASTCoordinateSystems.doc
1/13/2025

The following are transformation equations defining the angular orientation of each coordinate system inherent in FAST.

Before providing these, it is useful to discuss the transformation equation relating coordinate system $\boldsymbol{x}$ to coordinate system $\boldsymbol{X}$ where $\boldsymbol{x}$ (with orthogonal axes $\boldsymbol{x_1}$, $\boldsymbol{x_2}$, and $\boldsymbol{x_3}$) is the coordinate system resulting from three rotations (θ_1, θ_2, and θ_3) about the orthogonal axes ($\boldsymbol{X_1}$, $\boldsymbol{X_2}$, and $\boldsymbol{X_3}$) of coordinate system $\boldsymbol{X}$. With all rotation angles assumed to be small, the order of rotations does not matter and Euler angles do not need to be used. Instead, what we want, is a transformation equation that is consistent with classical Bernoulli-Euler beam theory (which assumes small rotations). The correct transformation equation is:

$$\begin{Bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \boldsymbol{x_3} \end{Bmatrix} \approx \underbrace{\begin{bmatrix} 1 & \theta_3 & -\theta_2 \\ -\theta_3 & 1 & \theta_1 \\ \theta_2 & -\theta_1 & 1 \end{bmatrix}}_{[A]} \begin{Bmatrix} \boldsymbol{X_1} \\ \boldsymbol{X_2} \\ \boldsymbol{X_3} \end{Bmatrix},$$

where $[A]$ is referred to as the Bernoulli-Euler transformation matrix in this work. The approximation symbol (≈) is used in place of an equals symbol (=) in the above expression since $[A]$ is not orthonormal, which implies that the resulting $\boldsymbol{x}$ from this expression is not made up of a set of mutually orthogonal axes (all transformation matrices between sets of mutually orthogonal axes must be orthonormal). So it is evident that in place of $[A]$, what we want is the closest orthonormal matrix to $[A]$, which is referred to as $[TransMat]$ in this work. From linear algebra, we know that the closest orthonormal matrix to $[A]$ in the Frobenius Norm sense is:

$$[TransMat] = [U][V]^T,$$

where the columns of $[U]$ contain the eigenvectors of $[A][A]^T$ and the columns of $[V]$ contain the eigenvectors of $[A]^T[A]$. This result follows directly from the Singular Value Decomposition (SVD) of $[A]$:

$$[A] = [U][\Sigma][V]^T,$$

where $[\Sigma]$ is a diagonal matrix containing the singular values of $[A]$, which are $\sqrt{eigenvalues\ of\ [A][A]^T} = \sqrt{eigenvalues\ of\ [A]^T[A]}$.

The algebraic form of the resulting transformation matrix is:

$$\begin{Bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \boldsymbol{x_3} \end{Bmatrix} = \underbrace{\begin{bmatrix} \dfrac{\theta_1^2\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}+\theta_2^2+\theta_3^2}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} & \dfrac{\theta_3\left(\theta_1^2+\theta_2^2+\theta_3^2\right)+\theta_1\theta_2\left(\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}-1\right)}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} & \dfrac{-\theta_2\left(\theta_1^2+\theta_2^2+\theta_3^2\right)+\theta_1\theta_3\left(\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}-1\right)}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} \\ \dfrac{-\theta_3\left(\theta_1^2+\theta_2^2+\theta_3^2\right)+\theta_1\theta_2\left(\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}-1\right)}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} & \dfrac{\theta_1^2+\theta_2^2\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}+\theta_3^2}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} & \dfrac{\theta_1\left(\theta_1^2+\theta_2^2+\theta_3^2\right)+\theta_2\theta_3\left(\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}-1\right)}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} \\ \dfrac{\theta_2\left(\theta_1^2+\theta_2^2+\theta_3^2\right)+\theta_1\theta_3\left(\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}-1\right)}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} & \dfrac{-\theta_1\left(\theta_1^2+\theta_2^2+\theta_3^2\right)+\theta_2\theta_3\left(\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}-1\right)}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} & \dfrac{\theta_1^2+\theta_2^2+\theta_3^2\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}}{\left(\theta_1^2+\theta_2^2+\theta_3^2\right)\sqrt{1+\theta_1^2+\theta_2^2+\theta_3^2}} \end{bmatrix}}_{[TransMat]} \begin{Bmatrix} \boldsymbol{X_1} \\ \boldsymbol{X_2} \\ \boldsymbol{X_3} \end{Bmatrix}$$

This was derived symbolically by J. Jonkman by computing $[U][V]^T$ by hand with verification in Mathematica.

Tower Base / Platform Coordinate System

$$\begin{Bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \\ \boldsymbol{a_3} \end{Bmatrix} = \left[TransMat\left(\theta_1 = q_R, \theta_2 = q_Y, \theta_3 = -q_P\right)\right] \begin{Bmatrix} \boldsymbol{z_1} \\ \boldsymbol{z_2} \\ \boldsymbol{z_3} \end{Bmatrix}$$

Tower Element-Fixed Coordinate System

$$\begin{Bmatrix} \boldsymbol{t_1}(h) \\ \boldsymbol{t_2}(h) \\ \boldsymbol{t_3}(h) \end{Bmatrix} = \left[TransMat\left(\theta_1 = \theta_{SS}(h), \theta_2 = 0, \theta_3 = \theta_{FA}(h)\right)\right] \begin{Bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \\ \boldsymbol{a_3} \end{Bmatrix}$$

where,

$$\theta_{FA}(h) = -\left[\frac{d\phi_1^{TFA}(h)}{dh} q_{TFA1} + \frac{d\phi_2^{TFA}(h)}{dh} q_{TFA2}\right]$$ and $$\theta_{SS}(h) = \left[\frac{d\phi_1^{TSS}(h)}{dh} q_{TSS1} + \frac{d\phi_2^{TSS}(h)}{dh} q_{TSS2}\right]$$

Jason Jonkman
FASTCoordinateSystems.doc
1/13/2025

Tower-Top / Base Plate Coordinate System

$$\begin{Bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \\ \boldsymbol{b_3} \end{Bmatrix} = \left[TransMat\left(\theta_1 = \theta_{SS}\left(TwrFlexL\right), \theta_2 = 0, \theta_3 = \theta_{FA}\left(TwrFlexL\right) \right) \right] \begin{Bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \\ \boldsymbol{a_3} \end{Bmatrix}$$

where,

$$\theta_{FA}\left(TwrFlexL\right) = -\left[\left. \frac{d\phi_1^{TFA}\left(h\right)}{dh} \right|_{h=TwrFlexL} q_{TFA1} + \left. \frac{d\phi_2^{TFA}\left(h\right)}{dh} \right|_{h=TwrFlexL} q_{TFA2} \right]$$ and $$\theta_{SS}\left(TwrFlexL\right) = \left[\left. \frac{d\phi_1^{TSS}\left(h\right)}{dh} \right|_{h=TwrFlexL} q_{TSS1} + \left. \frac{d\phi_2^{TSS}\left(h\right)}{dh} \right|_{h=TwrFlexL} q_{TSS2} \right]$$

Nacelle / Yaw Coordinate System

$$\begin{Bmatrix} \boldsymbol{d_1} \\ \boldsymbol{d_2} \\ \boldsymbol{d_3} \end{Bmatrix} = \begin{bmatrix} cos\left(q_{Yaw}\right) & 0 & -sin\left(q_{Yaw}\right) \\ 0 & 1 & 0 \\ sin\left(q_{Yaw}\right) & 0 & cos\left(q_{Yaw}\right) \end{bmatrix} \begin{Bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \\ \boldsymbol{b_3} \end{Bmatrix}$$

Rotor-Furl Coordinate System

$$\begin{Bmatrix} \boldsymbol{rf_1} \\ \boldsymbol{rf_2} \\ \boldsymbol{rf_3} \end{Bmatrix} = \begin{bmatrix} \begin{gathered} \left[1 - cos^2\left(RFrlSkew\right) cos^2\left(RFrlTilt\right)\right] cos\left(q_{RFrl}\right) \\ + cos^2\left(RFrlSkew\right) cos^2\left(RFrlTilt\right) \end{gathered} & \begin{gathered} cos\left(RFrlSkew\right) cos\left(RFrlTilt\right) sin\left(RFrlTilt\right)\left[1 - cos\left(q_{RFrl}\right)\right] \\ - sin\left(RFrlSkew\right) cos\left(RFrlTilt\right) sin\left(q_{RFrl}\right) \end{gathered} & \begin{gathered} cos\left(RFrlSkew\right) sin\left(RFrlSkew\right) cos^2\left(RFrlTilt\right)\left[cos\left(q_{RFrl}\right) - 1\right] \\ - sin\left(RFrlTilt\right) sin\left(q_{RFrl}\right) \end{gathered} \\ \begin{gathered} cos\left(RFrlSkew\right) cos\left(RFrlTilt\right) sin\left(RFrlTilt\right)\left[1 - cos\left(q_{RFrl}\right)\right] \\ + sin\left(RFrlSkew\right) cos\left(RFrlTilt\right) sin\left(q_{RFrl}\right) \end{gathered} & cos^2\left(RFrlTilt\right) cos\left(q_{RFrl}\right) + sin^2\left(RFrlTilt\right) & \begin{gathered} sin\left(RFrlSkew\right) cos\left(RFrlTilt\right) sin\left(RFrlTilt\right)\left[cos\left(q_{RFrl}\right) - 1\right] \\ + cos\left(RFrlSkew\right) cos\left(RFrlTilt\right) sin\left(q_{RFrl}\right) \end{gathered} \\ \begin{gathered} cos\left(RFrlSkew\right) sin\left(RFrlSkew\right) cos^2\left(RFrlTilt\right)\left[cos\left(q_{RFrl}\right) - 1\right] \\ + sin\left(RFrlTilt\right) sin\left(q_{RFrl}\right) \end{gathered} & \begin{gathered} sin\left(RFrlSkew\right) cos\left(RFrlTilt\right) sin\left(RFrlTilt\right)\left[cos\left(q_{RFrl}\right) - 1\right] \\ - cos\left(RFrlSkew\right) cos\left(RFrlTilt\right) sin\left(q_{RFrl}\right) \end{gathered} & \begin{gathered} \left[1 - sin^2\left(RFrlSkew\right) cos^2\left(RFrlTilt\right)\right] cos\left(q_{RFrl}\right) \\ + sin^2\left(RFrlSkew\right) cos^2\left(RFrlTilt\right) \end{gathered} \end{bmatrix} \begin{Bmatrix} \boldsymbol{d_1} \\ \boldsymbol{d_2} \\ \boldsymbol{d_3} \end{Bmatrix}$$

Shaft Coordinate System

$$\begin{Bmatrix} \boldsymbol{c_1} \\ \boldsymbol{c_2} \\ \boldsymbol{c_3} \end{Bmatrix} = \begin{bmatrix} cos\left(ShftSkew\right) cos\left(ShftTilt\right) & sin\left(ShftTilt\right) & -sin\left(ShftSkew\right) cos\left(ShftTilt\right) \\ -cos\left(ShftSkew\right) sin\left(ShftTilt\right) & cos\left(ShftTilt\right) & sin\left(ShftSkew\right) sin\left(ShftTilt\right) \\ sin\left(ShftSkew\right) & 0 & cos\left(ShftSkew\right) \end{bmatrix} \begin{Bmatrix} \boldsymbol{rf_1} \\ \boldsymbol{rf_2} \\ \boldsymbol{rf_3} \end{Bmatrix}$$

Azimuth Coordinate System

$$\begin{Bmatrix} \boldsymbol{e_1} \\ \boldsymbol{e_2} \\ \boldsymbol{e_3} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\left(q_{DrTr} + q_{GeAz}\right) & \sin\left(q_{DrTr} + q_{GeAz}\right) \\ 0 & -\sin\left(q_{DrTr} + q_{GeAz}\right) & \cos\left(q_{DrTr} + q_{GeAz}\right) \end{bmatrix} \begin{Bmatrix} \boldsymbol{c_1} \\ \boldsymbol{c_2} \\ \boldsymbol{c_3} \end{Bmatrix}$$

Teeter Coordinate System

$$\begin{Bmatrix} \boldsymbol{f_1} \\ \boldsymbol{f_2} \\ \boldsymbol{f_3} \end{Bmatrix} = \begin{bmatrix} \cos\left(q_{Teet}\right) & 0 & -\sin\left(q_{Teet}\right) \\ 0 & 1 & 0 \\ \sin\left(q_{Teet}\right) & 0 & \cos\left(q_{Teet}\right) \end{bmatrix} \begin{Bmatrix} \boldsymbol{e_1} \\ \boldsymbol{e_2} \\ \boldsymbol{e_3} \end{Bmatrix}$$

Hub / Delta-3 Coordinate System

$$\begin{Bmatrix} \boldsymbol{g_1} \\ \boldsymbol{g_2} \\ \boldsymbol{g_3} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\left(Delta3\right) & \sin\left(Delta3\right) \\ 0 & -\sin\left(Delta3\right) & \cos\left(Delta3\right) \end{bmatrix} \begin{Bmatrix} \boldsymbol{f_1} \\ \boldsymbol{f_2} \\ \boldsymbol{f_3} \end{Bmatrix}$$

Hub (Prime) Coordinate System

$$\begin{Bmatrix} \boldsymbol{g'^{B1}_1} \\ \boldsymbol{g'^{B1}_2} \\ \boldsymbol{g'^{B1}_3} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \boldsymbol{g_1} \\ \boldsymbol{g_2} \\ \boldsymbol{g_3} \end{Bmatrix}$$

The equation for $\boldsymbol{g}'^{B2}$ of blade 2 is similar.

Coned Coordinate System

$$\begin{Bmatrix} i_1^{B1} \\ i_2^{B1} \\ i_3^{B1} \end{Bmatrix} = \begin{bmatrix} \cos\left[PreCone(1)\right] & 0 & -\sin\left[PreCone(1)\right] \\ 0 & 1 & 0 \\ \sin\left[PreCone(1)\right] & 0 & \cos\left[PreCone(1)\right] \end{bmatrix} \begin{Bmatrix} g'^{B1}_1 \\ g'^{B1}_2 \\ g'^{B1}_3 \end{Bmatrix}$$

The equation for $\boldsymbol{i}^{B2}$ is similar.

Blade / Pitched Coordinate System

$$\begin{Bmatrix} j_1^{B1} \\ j_2^{B1} \\ j_3^{B1} \end{Bmatrix} = \begin{bmatrix} \cos\left[BlPitch(1)\right] & -\sin\left[BlPitch(1)\right] & 0 \\ \sin\left[BlPitch(1)\right] & \cos\left[BlPitch(1)\right] & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} i_1^{B1} \\ i_2^{B1} \\ i_3^{B1} \end{Bmatrix}$$

The equation for $\boldsymbol{j}^{B2}$ is similar.

Blade Coordinate System Aligned with Local Structural Axes (not element fixed)

$$\begin{Bmatrix} Lj_1^{B1}(r) \\ Lj_2^{B1}(r) \\ Lj_3^{B1}(r) \end{Bmatrix} = \begin{bmatrix} \cos\left[\theta_S^{B1}(r)\right] & -\sin\left[\theta_S^{B1}(r)\right] & 0 \\ \sin\left[\theta_S^{B1}(r)\right] & \cos\left[\theta_S^{B1}(r)\right] & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} i_1^{B1} \\ i_2^{B1} \\ i_3^{B1} \end{Bmatrix}$$

The equation for $\boldsymbol{Lj}^{B2}(r)$ is similar.

Blade Element-Fixed Coordinate System Aligned with Local Structural Axes

$$\begin{Bmatrix} n_1^{B1}(r) \\ n_2^{B1}(r) \\ n_3^{B1}(r) \end{Bmatrix} = \left[TransMat\left(\theta_1 = \theta_x^{B1}(r), \theta_2 = \theta_y^{B1}(r), \theta_3 = 0\right)\right] \begin{Bmatrix} Lj_1^{B1}(r) \\ Lj_2^{B1}(r) \\ Lj_3^{B1}(r) \end{Bmatrix}$$

where,

$$\theta_x^{B1}(r) = cos\left[\theta_S^{B1}(r)\right]\theta_{IP}^{B1}(r) - sin\left[\theta_S^{B1}(r)\right]\theta_{OoP}^{B1}(r),$$

$$\theta_y^{B1}(r) = sin\left[\theta_S^{B1}(r)\right]\theta_{IP}^{B1}(r) + cos\left[\theta_S^{B1}(r)\right]\theta_{OoP}^{B1}(r),$$ and

$$\theta_{IP}^{B1}(r) = -\left[\frac{d\psi_1^{B1}(r)}{dr}q_{B1F1} + \frac{d\psi_2^{B1}(r)}{dr}q_{B1F2} + \frac{d\psi_3^{B1}(r)}{dr}q_{B1E1}\right],$$

$$\theta_{OoP}^{B1}(r) = \left[\frac{d\phi_1^{B1}(r)}{dr}q_{B1F1} + \frac{d\phi_2^{B1}(r)}{dr}q_{B1F2} + \frac{d\phi_3^{B1}(r)}{dr}q_{B1E1}\right]$$

The equation for $\boldsymbol{n}^{B2}(r)$ is similar.

Blade Element-Fixed Coordinate System Used for Calculating and Returning Aerodynamic Loads

This coordinate system is coincident with $\boldsymbol{i}^{B1}$ when the blade is undeflected.

$$\begin{Bmatrix} \boldsymbol{m}_1^{B1}(r) \\ \boldsymbol{m}_2^{B1}(r) \\ \boldsymbol{m}_3^{B1}(r) \end{Bmatrix} = \begin{bmatrix} cos\left[BlPitch(1)+\theta_S^{B1}(r)\right] & sin\left[BlPitch(1)+\theta_S^{B1}(r)\right] & 0 \\ -sin\left[BlPitch(1)+\theta_S^{B1}(r)\right] & cos\left[BlPitch(1)+\theta_S^{B1}(r)\right] & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \boldsymbol{n}_1^{B1}(r) \\ \boldsymbol{n}_2^{B1}(r) \\ \boldsymbol{n}_3^{B1}(r) \end{Bmatrix}$$

The equation for $\boldsymbol{m}^{B2}(r)$ is similar.

Blade Element-Fixed Coordinate System Aligned with Local Aerodynamic Axes (i.e., chordline) / Trailing Edge Coordinate System

$$\begin{Bmatrix} \boldsymbol{te}_1^{B1}(r) \\ \boldsymbol{te}_2^{B1}(r) \\ \boldsymbol{te}_3^{B1}(r) \end{Bmatrix} = \begin{bmatrix} cos\left[BlPitch(1)+\theta_A^{B1}(r)\right] & -sin\left[BlPitch(1)+\theta_A^{B1}(r)\right] & 0 \\ sin\left[BlPitch(1)+\theta_A^{B1}(r)\right] & cos\left[BlPitch(1)+\theta_A^{B1}(r)\right] & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \boldsymbol{m}_1^{B1}(r) \\ \boldsymbol{m}_2^{B1}(r) \\ \boldsymbol{m}_3^{B1}(r) \end{Bmatrix}$$

The equation for $\boldsymbol{te}^{B2}(r)$ is similar.

Jason Jonkman

Tail-Furl Coordinate System

$$\begin{Bmatrix} \boldsymbol{tf_1} \\ \boldsymbol{tf_2} \\ \boldsymbol{tf_3} \end{Bmatrix} = \begin{bmatrix} \begin{matrix} \left[1 - cos^2(TFrlSkew)cos^2(TFrlTilt)\right]cos(q_{TFrl}) \\ +cos^2(TFrlSkew)cos^2(TFrlTilt) \end{matrix} & \begin{matrix} cos(TFrlSkew)cos(TFrlTilt)sin(TFrlTilt)\left[1 - cos(q_{TFrl})\right] \\ - sin(TFrlSkew)cos(TFrlTilt)sin(q_{TFrl}) \end{matrix} & \begin{matrix} cos(TFrlSkew)sin(TFrlSkew)cos^2(TFrlTilt)\left[cos(q_{TFrl}) - 1\right] \\ - sin(TFrlTilt)sin(q_{TFrl}) \end{matrix} \\ \begin{matrix} cos(TFrlSkew)cos(TFrlTilt)sin(TFrlTilt)\left[1 - cos(q_{TFrl})\right] \\ + sin(TFrlSkew)cos(TFrlTilt)sin(q_{TFrl}) \end{matrix} & cos^2(TFrlTilt)cos(q_{TFrl}) + sin^2(TFrlTilt) & \begin{matrix} sin(TFrlSkew)cos(TFrlTilt)sin(TFrlTilt)\left[cos(q_{TFrl}) - 1\right] \\ + cos(TFrlSkew)cos(TFrlTilt)sin(q_{TFrl}) \end{matrix} \\ \begin{matrix} cos(TFrlSkew)sin(TFrlSkew)cos^2(TFrlTilt)\left[cos(q_{TFrl}) - 1\right] \\ + sin(TFrlTilt)sin(q_{TFrl}) \end{matrix} & \begin{matrix} sin(TFrlSkew)cos(TFrlTilt)sin(TFrlTilt)\left[cos(q_{TFrl}) - 1\right] \\ - cos(TFrlSkew)cos(TFrlTilt)sin(q_{TFrl}) \end{matrix} & \begin{matrix} \left[1 - sin^2(TFrlSkew)cos^2(TFrlTilt)\right]cos(q_{TFrl}) \\ +sin^2(TFrlSkew)cos^2(TFrlTilt) \end{matrix} \end{bmatrix} \begin{Bmatrix} \boldsymbol{d_1} \\ \boldsymbol{d_2} \\ \boldsymbol{d_3} \end{Bmatrix}$$

Tail Fin Coordinate System

$$\begin{Bmatrix} \boldsymbol{p_1} \\ \boldsymbol{p_2} \\ \boldsymbol{p_3} \end{Bmatrix} = \begin{bmatrix} cos(TFinSkew)cos(TFinTilt) & sin(TFinTilt) & - sin(TFinSkew)cos(TFinTilt) \\ \begin{matrix} sin(TFinSkew)sin(TFinBank) \\ - cos(TFinSkew)sin(TFintTilt)cos(TFinBank) \end{matrix} & cos(TFinTilt)cos(TFinBank) & \begin{matrix} cos(TFinSkew)sin(TFinBank) \\ + sin(TFintSkew)sin(TFinTilt)cos(TFinBank) \end{matrix} \\ \begin{matrix} sin(TFinSkew)cos(TFinBank) \\ + cos(TFinSkew)sin(TFintTilt)sin(TFinBank) \end{matrix} & - cos(TFinTilt)sin(TFinBank) & \begin{matrix} cos(TFinSkew)cos(TFinBank) \\ - sin(TFintSkew)sin(TFinTilt)sin(TFinBank) \end{matrix} \end{bmatrix} \begin{Bmatrix} \boldsymbol{tf_1} \\ \boldsymbol{tf_2} \\ \boldsymbol{tf_3} \end{Bmatrix}$$