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The following are derivations of the entire equations of motion used in FAST for a 2-bladed turbine configuration. The various portions of the equations of motion are organized according to their source. The equations of motion of a 3-bladed turbine are very similar.

By a direct result of Newton's laws of motion, Kane's equations of motion for a simple holonomic system with 22 DOFs can be stated as follows (Kane and Levinson, 1985):

$$F_r + F_r^* = 0 \quad (r = 1,2,\ldots,22)$$

where, for a set of w rigid bodies characterized by reference frame N_i and center of mass point X_i:

the *generalized active forces* are: $$F_r = \sum_{i=1}^{w} {}^{E}\boldsymbol{v}_r^{X_i} \cdot \boldsymbol{F}^{X_i} + {}^{E}\boldsymbol{\omega}_r^{N_i} \cdot \boldsymbol{M}^{N_i} \quad (r = 1,2,\ldots,22)$$

and the *generalized inertia forces* are: $$F_r^* = \sum_{i=1}^{w} {}^{E}\boldsymbol{v}_r^{X_i} \cdot \left(-m^{N_i}\, {}^{E}\boldsymbol{a}^{X_i}\right) + {}^{E}\boldsymbol{\omega}_r^{N_i} \cdot \left(-{}^{E}\dot{\boldsymbol{H}}^{N_i}\right) \quad (r = 1,2,\ldots,22)$$

where it is assumed that for each rigid body N_i, the active forces are applied at the center of mass point X_i. The time derivative of the angular momentum of rigid body N_i about its center of mass X_i in the inertial frame can be found as follows:

$${}^{E}\dot{\boldsymbol{H}}^{N_i} = \begin{cases} \left(\dot{\boldsymbol{H}}^{N_i}\right)' + {}^{E}\boldsymbol{\omega}^{N_i} \times {}^{E}\boldsymbol{H}^{N_i} \\ or \\ \bar{\bar{\boldsymbol{I}}}^{N_i} \cdot {}^{E}\boldsymbol{\alpha}^{N_i} + {}^{E}\boldsymbol{\omega}^{N_i} \times \bar{\bar{\boldsymbol{I}}}^{N_i} \cdot {}^{E}\boldsymbol{\omega}^{N_i} \end{cases}$$

For the wind turbine modeled in FAST, the mass of the platform, tower, yaw bearing, nacelle, structure that furls with the rotor, hub, blades, generator, and tail contribute to the total generalized inertia forces as follows:

$$F_r^* = F_r^*\big|_X + F_r^*\big|_T + F_r^*\big|_N + F_r^*\big|_R + F_r^*\big|_H + F_r^*\big|_{B1} + F_r^*\big|_{B2} + F_r^*\big|_G + F_r^*\big|_A \quad (r = 1,2,\ldots,22)$$

Generalized active forces are the result of forces applied directly on the wind turbine system, forces that ensure constraint relationships between the various rigid bodies, and internal forces within flexible members. Forces applied directly on the wind turbine system include aerodynamic forces acting on the blades, tower, and tail fin; hydrostatic, hydrodynamic, mooring and/or foundation elasticity and damping forces, including added mass effects, acting on the platform; gravitational forces acting on the platform, tower, yaw bearing, nacelle, structure that furls with the rotor, hub, blades, tip brakes, and tail; generator torque; HSS brake; and gearbox friction forces. Forces that enforce constraint relationships between the various rigid bodies include springs and dampers for yaw, rotor-furl, teeter, and tail-furl (the simple workless constraint forces, for example frictionless pins or rigid connections, do not contribute to the generalized active forces). Internal forces within flexible members include elasticity and damping in the tower, blades, and drivetrain. Thus,

$$\begin{aligned} F_r = {} & F_r\big|_{AeroT} + F_r\big|_{AeroB1} + F_r\big|_{AeroB2} + F_r\big|_{AeroA} + F_r\big|_{HydroX} + F_r\big|_{GravX} + F_r\big|_{GravT} + F_r\big|_{GravN} + F_r\big|_{GravR} + F_r\big|_{GravH} + F_r\big|_{GravB1} + F_r\big|_{GravB2} + F_r\big|_{GravA} \\ & + F_r\big|_{SpringYaw} + F_r\big|_{DampYaw} + F_r\big|_{SpringRF} + F_r\big|_{DampRF} + F_r\big|_{SpringTeet} + F_r\big|_{DampTeet} + F_r\big|_{SpringTF} + F_r\big|_{DampTF} + F_r\big|_{Gen} + F_r\big|_{Brake} + F_r\big|_{GBFric} \qquad \left(r = 1,2,\ldots,22\right) \\ & + F_r\big|_{ElasticT} + F_r\big|_{DampT} + F_r\big|_{ElasticB1} + F_r\big|_{DampB1} + F_r\big|_{ElasticB2} + F_r\big|_{DampB2} + F_r\big|_{ElasticDrive} + F_r\big|_{DampDrive} \end{aligned}$$

Kane’s equations of motion can be written in matrix form as follows:

$$\left[C\left(q,t\right)\right]\left\{\ddot{q}\right\} + \left\{f\left(\dot{q},q,t\right)\right\} = \left\{0\right\} \qquad \text{or,} \qquad \left[C\left(q,t\right)\right]\left\{\ddot{q}\right\} = \left\{-f\left(\dot{q},q,t\right)\right\}$$

Platform:

The rigid lump mass of the platform brings about generalized inertia forces and generalized active forces associated with platform weight, hydrodynamics and hydrostatics, and mooring line and/or foundation elasticity and damping, including added mass effects.

$$F_r^*\Big|_X = {}^E\boldsymbol{v}_r^Y \cdot \left(-m^X\, {}^E\boldsymbol{a}^Y\right) + {}^E\boldsymbol{\omega}_r^X \cdot \left(-{}^E\dot{\boldsymbol{H}}^X\right) \quad (r = 1,2,\ldots,22) \qquad \text{where} \qquad m^X = PtfmMass$$

Thus,

$$F_r^*\Big|_X = {}^E\boldsymbol{v}_r^Y \cdot \left(-m^X\, {}^E\boldsymbol{a}^Y\right) + {}^E\boldsymbol{\omega}_r^X \cdot \left(-\bar{\bar{\boldsymbol{I}}}^X \cdot {}^E\boldsymbol{\alpha}^X - {}^E\boldsymbol{\omega}^X \times \bar{\bar{\boldsymbol{I}}}^X \cdot {}^E\boldsymbol{\omega}^X\right) \quad (r = 1,2,\ldots,22)$$

where $\bar{\bar{\boldsymbol{I}}}^X = PtfmRIner\boldsymbol{a}_1\boldsymbol{a}_1 + PtfmYIner\boldsymbol{a}_2\boldsymbol{a}_2 + PtfmPIner\boldsymbol{a}_3\boldsymbol{a}_3$

Or,

$$F_r^*\Big|_X = {}^E\boldsymbol{v}_r^Y \cdot \left(-m^X \left\{\left(\sum_{i=1}^{6} {}^E\boldsymbol{v}_i^Y \ddot{q}_i\right) + \left[\sum_{i=4}^{6} \frac{d}{dt}\left({}^E\boldsymbol{v}_i^Y\right)\dot{q}_i\right]\right\}\right) + {}^E\boldsymbol{\omega}_r^X \cdot \left[-\bar{\bar{\boldsymbol{I}}}^X \cdot \left(\sum_{i=4}^{6} {}^E\boldsymbol{\omega}_i^X \ddot{q}_i\right) - {}^E\boldsymbol{\omega}^X \times \bar{\bar{\boldsymbol{I}}}^X \cdot {}^E\boldsymbol{\omega}^X\right] \quad (r = 1,2,\ldots,6)$$

Thus,

$$\left[C(q,t)\right]\Big|_X (Row,Col) = m^X\, {}^E\boldsymbol{v}_{Row}^Y \cdot {}^E\boldsymbol{v}_{Col}^Y + {}^E\boldsymbol{\omega}_{Row}^X \cdot \bar{\bar{\boldsymbol{I}}}^X \cdot {}^E\boldsymbol{\omega}_{Col}^X \quad (Row,Col = 1,2,\ldots,6)$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_X (Row) = -m^X\, {}^E\boldsymbol{v}_{Row}^Y \cdot \left[\sum_{i=4}^{6} \frac{d}{dt}\left({}^E\boldsymbol{v}_i^Y\right)\dot{q}_i\right] - {}^E\boldsymbol{\omega}_{Row}^X \cdot \left({}^E\boldsymbol{\omega}^X \times \bar{\bar{\boldsymbol{I}}}^X \cdot {}^E\boldsymbol{\omega}^X\right) \quad (Row = 1,2,\ldots,6)$$

$$F_r\Big|_{GravX} = {}^E\boldsymbol{v}_r^Y \cdot \left(-m^X g \boldsymbol{z}_2\right) \quad (r = 3,4,\ldots,6) \qquad \text{where} \qquad g = Gravity$$

Thus,

$$\left[C(q,t)\right]\Big|_{GravX} = 0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{GravX} (Row) = -m^X g\, {}^E\boldsymbol{v}_{Row}^Y \cdot \boldsymbol{z}_2 \quad (Row = 3,4,\ldots,6)$$

$$F_r\Big|_{HydroX} = {}^E\boldsymbol{v}_r^Y \cdot \boldsymbol{F}_{Hydro}^Y + {}^E\boldsymbol{\omega}_r^X \cdot \boldsymbol{M}_{Hydro}^{X@Y} \quad (r = 1,2,\ldots,22)$$

where the equivalent loads acting at the platform center of mass are related to $\boldsymbol{F}_{Hydro}^Z$ and $\boldsymbol{M}_{Hydro}^{X@Z}$ because the platform is rigid as follows:

$$\boldsymbol{F}_{Hydro}^Y = \boldsymbol{F}_{Hydro}^Z \qquad \text{and} \qquad \boldsymbol{M}_{Hydro}^{X@Y} = \boldsymbol{M}_{Hydro}^{X@Z} + \boldsymbol{r}^{YZ} \times \boldsymbol{F}_{Hydro}^Z = \boldsymbol{M}_{Hydro}^{X@Z} - \boldsymbol{r}^{ZY} \times \boldsymbol{F}_{Hydro}^Z$$

since $\boldsymbol{r}^{YZ} = -\boldsymbol{r}^{ZY}$.

But since ${}^{E}\boldsymbol{v}_{r}^{Y} = {}^{E}\boldsymbol{v}_{r}^{Z} + {}^{E}\boldsymbol{\omega}_{r}^{X} \times \boldsymbol{r}^{ZY}$, this generalized active force can be expanded to:

$$F_r\big|_{HydroX} = \left({}^{E}\boldsymbol{v}_{r}^{Z} + {}^{E}\boldsymbol{\omega}_{r}^{X} \times \boldsymbol{r}^{ZY} \right) \cdot \boldsymbol{F}_{Hydro}^{Z} + {}^{E}\boldsymbol{\omega}_{r}^{X} \cdot \left(\boldsymbol{M}_{Hydro}^{X@Z} - \boldsymbol{r}^{ZY} \times \boldsymbol{F}_{Hydro}^{Z} \right) \quad (r = 1,2,\ldots,22)$$

Now applying the cyclic permutation law of the scalar triple product, the generalized active force simplifies to:

$$F_r\big|_{HydroX} = {}^{E}\boldsymbol{v}_{r}^{Z} \cdot \boldsymbol{F}_{Hydro}^{Z} + {}^{E}\boldsymbol{\omega}_{r}^{X} \cdot \boldsymbol{M}_{Hydro}^{X@Z} \quad (r = 1,2,\ldots,6)$$

But,

$$\boldsymbol{F}_{Hydro}^{Z} = \left(\sum_{j=1}^{6} \boldsymbol{F}_{Hydro_j}^{Z} \ddot{q}_i \right) + \boldsymbol{F}_{Hydro_t}^{Z} \quad \text{and} \quad \boldsymbol{M}_{Hydro}^{X@Z} = \left(\sum_{j=1}^{6} \boldsymbol{M}_{Hydro_j}^{X@Z} \ddot{q}_i \right) + \boldsymbol{M}_{Hydro_t}^{X@Z}$$

where,

$$\boldsymbol{F}_{Hydro_j}^{Z} = -\left(\sum_{i=1}^{3} a_{ij} \, {}^{E}\boldsymbol{v}_{i}^{Z} \right) \quad (j = 1,2,\ldots,6) \quad \text{and} \quad \boldsymbol{M}_{Hydro_j}^{X@Z} = -\left(\sum_{i=4}^{6} a_{ij} \, {}^{E}\boldsymbol{\omega}_{i}^{X} \right) \quad (j = 1,2,\ldots,6)$$

with $a_{ij} \quad (i,j = 1,2,\ldots,6)$ being the added mass coefficients (or equivalently, $[a]$ being the added mass matrix), $\boldsymbol{F}_{Hydro_j}^{Z} \quad (j = 1,2,\ldots,6)$ and $\boldsymbol{M}_{Hydro_j}^{X@Z} \quad (j = 1,2,\ldots,6)$ being the partial hydrodynamic added mass forces and moments, and $\boldsymbol{F}_{Hydro_t}^{Z}$ and $\boldsymbol{M}_{Hydro_t}^{X@Z}$ being the contributions to $\boldsymbol{F}_{Hydro}^{Z}$ and $\boldsymbol{M}_{Hydro}^{X@Z}$ that don't depend on platform accelerations.

Thus,

$$F_r\big|_{HydroX} = {}^{E}\boldsymbol{v}_{r}^{Z} \cdot \left[\left(\sum_{j=1}^{6} \boldsymbol{F}_{Hydro_j}^{Z} \ddot{q}_i \right) + \boldsymbol{F}_{Hydro_t}^{Z} \right] + {}^{E}\boldsymbol{\omega}_{r}^{X} \cdot \left[\left(\sum_{j=1}^{6} \boldsymbol{M}_{Hydro_j}^{X@Z} \ddot{q}_i \right) + \boldsymbol{M}_{Hydro_t}^{X@Z} \right] \quad (r = 1,2,\ldots,6)$$

and

$$\left[C(q,t) \right]\Big|_{HydroX} (Row,Col) = [a](Row,Col) = -{}^{E}\boldsymbol{v}_{Row}^{Z} \cdot \boldsymbol{F}_{Hydro_{Col}}^{Z} - {}^{E}\boldsymbol{\omega}_{Row}^{X} \cdot \boldsymbol{M}_{Hydro_{Col}}^{X@Z} \quad (Row,Col = 1,2,\ldots,6)$$

$$\left\{ -f(\dot{q},q,t) \right\}\Big|_{HydroX} (Row) = {}^{E}\boldsymbol{v}_{Row}^{Z} \cdot \boldsymbol{F}_{Hydro_t}^{Z} + {}^{E}\boldsymbol{\omega}_{Row}^{X} \cdot \boldsymbol{M}_{Hydro_t}^{X@Z} \quad (Row = 1,2,\ldots,6)$$

Tower:

The distributed properties of the tower bring about generalized inertia forces and generalized active forces associated with tower elasticity, tower damping, tower aerodynamics, and tower weight. Note that I eliminated the tower mass tuners, since it is redundant to have both mass and stiffness tuners when trying to tune tower frequencies (to tune the frequencies for individual modes, all that is needed is to tune the mass or the stiffness for the individual modes, but not both). Note also that I eliminated the tower stiffness tuner's effects on the gravitational destiffening loads. It is also beneficial to eliminate the tower mass tuners because the tower mass density is needed to compute the tower base loads and thus these tuners affect the tower base loads directly—this makes the form of the tower base load equations considerably more complex and considerably less intuitive. Since the tower elastic stiffness does not directly influence the tower base loads in a fundamental way, the retention of the tower stiffness tuners is much more favorable than the retention of the tower mass tuners (recall that only one set of tuners needs to be retained in order to permit the user to match natural frequencies). The elimination of the tower stiffness tuner's effects on the gravitational destiffening was done for the same reason (i.e., the gravity loads directly affect the tower base loads, and thus, tower stiffness tuners make the form of the tower base load equations considerable more complex and considerably less intuitive). The fact that the gravitational destiffening of the tower is small compared to the overall stiffness of the tower is another reason this elimination of stiffness tuning effects should not be of significant concern.

$$F_r^*\Big|_T = -\int_0^{TwrFlexL} \mu^T(h)\,{}^E\boldsymbol{v}_r^T(h)\cdot{}^E\boldsymbol{a}^T(h)\,dh - YawBrMass\,{}^E\boldsymbol{v}_r^O\cdot{}^E\boldsymbol{a}^O \quad (r=1,2,\ldots,22) \qquad \text{where} \qquad \mu^T(h) = AdjTwMa\cdot TMassDen(h)$$

Or,

$$F_r^*\Big|_T = -\int_0^{TwrFlexL} \mu^T(h)\,{}^E\boldsymbol{v}_r^T(h)\cdot\left\{\left(\sum_{i=1}^{10}{}^E\boldsymbol{v}_i^T(h)\ddot{q}_i\right)+\left[\sum_{i=4}^{10}\frac{d}{dt}\left({}^E\boldsymbol{v}_i^T(h)\right)\dot{q}_i\right]\right\}dh - YawBrMass\,{}^E\boldsymbol{v}_r^O\cdot\left\{\left(\sum_{i=1}^{10}{}^E\boldsymbol{v}_i^O\ddot{q}_i\right)+\left[\sum_{i=4}^{10}\frac{d}{dt}\left({}^E\boldsymbol{v}_i^O\right)\dot{q}_i\right]\right\} \quad (r=1,2,\ldots,10)$$

Thus,

$$\left[C(q,t)\right]\Big|_T(Row,Col) = \int_0^{TwrFlexL} \mu^T(h)\,{}^E\boldsymbol{v}_{Row}^T(h)\cdot{}^E\boldsymbol{v}_{Col}^T(h)\,dh + YawBrMass\,{}^E\boldsymbol{v}_{Row}^O\cdot{}^E\boldsymbol{v}_{Col}^O \quad (Row,Col=1,2,\ldots,10)$$

$$\left\{-f(\dot{q},q,t)\right\}\Big|_T(Row) = -\int_0^{TwrFlexL} \mu^T(h)\,{}^E\boldsymbol{v}_{Row}^T(h)\cdot\left\{\sum_{i=4}^{10}\frac{d}{dt}\left[{}^E\boldsymbol{v}_i^T(h)\right]\dot{q}_i\right\}dh - YawBrMass\,{}^E\boldsymbol{v}_{Row}^O\cdot\left[\sum_{i=4}^{10}\frac{d}{dt}\left({}^E\boldsymbol{v}_i^O\right)\dot{q}_i\right] \quad (Row=1,2,\ldots,10)$$

$$F_r\Big|_{ElasticT} = -\frac{\partial V'^T}{\partial q_r} \quad (r=1,2,\ldots,22) \quad \text{So,} \quad F_r\Big|_{ElasticT} = \begin{cases} -k'^{TFA}_{11}\,q_{TFA1} - k'^{TFA}_{12}\,q_{TFA2} & for \quad r = TFA1 \\ -k'^{TSS}_{11}\,q_{TSS1} - k'^{TSS}_{12}\,q_{TSS2} & for \quad r = TSS1 \\ -k'^{TFA}_{21}\,q_{TFA1} - k'^{TFA}_{22}\,q_{TFA2} & for \quad r = TFA2 \\ -k'^{TSS}_{21}\,q_{TSS1} - k'^{TSS}_{22}\,q_{TSS2} & for \quad r = TSS2 \\ 0 & otherwise \end{cases}$$

where k'^{TFA}_{ij} and k'^{TSS}_{ij} are the generalized stiffnesses of the tower in the fore-aft and side-to-side directions respectively when gravitational destiffening effects are *not* included as follows:

$$k'^{TFA}_{ij} = \sqrt{FAStTunr(i)\,FAStTunr(j)} \int_0^{TwrFlexL} EI^{TFA}(h) \frac{d^2\phi_i^{TFA}(h)}{dh^2} \frac{d^2\phi_j^{TFA}(h)}{dh^2} dh \quad (i,j = 1,2) \text{ (which is symmetric)}$$

where $EI^{TFA}(h) = AdjFASt \cdot TwFAStif(h)$

and

$$k'^{TSS}_{ij} = \sqrt{SSStTunr(i)\,SSStTunr(j)} \int_0^{TwrFlexL} EI^{TSS}(h) \frac{d^2\phi_i^{TSS}(h)}{dh^2} \frac{d^2\phi_j^{TSS}(h)}{dh^2} dh \quad (i,j = 1,2) \quad \text{(which is symmetric)}$$

where $EI^{TSS}(h) = AdjSSSt \cdot TwSSStif(h)$

The coefficient in front of the integral in these generalized stiffnesses represents the individual modal stiffness tuning, which allows the user to vary the stiffness of the tower between the individual modes to permit better matching of tower frequencies. To be precise, the tuner coefficient only really makes sense when working with a generalized stiffness of a single mode (i.e., k'^{TFA}_{11}, k'^{TFA}_{22}, k'^{TSS}_{11}, or k'^{TSS}_{22}), in which case the coefficient for mode i is simply $FAStTunr(i)$ or $SSStTunr(i)$. However, since the cross-correlation elements of the generalized stiffness matrix will, in general, not vanish, the coefficient in the form above permits the tuning to apply to these terms in a consistent fashion.

Similarly, when using the Rayleigh damping technique where the damping is assumed proportional to the stiffness, then

$$F_r\Big|_{DampT} = \begin{cases} -\frac{\zeta_1^{TFA} k'^{TFA}_{11}}{\pi f'^{TFA}_1}\dot{q}_{TFA1} - \frac{\zeta_2^{TFA} k'^{TFA}_{12}}{\pi f'^{TFA}_2}\dot{q}_{TFA2} & for \quad r = TFA1 \\ -\frac{\zeta_1^{TSS} k'^{TSS}_{11}}{\pi f'^{TSS}_1}\dot{q}_{TSS1} - \frac{\zeta_2^{TSS} k'^{TSS}_{12}}{\pi f'^{TSS}_2}\dot{q}_{TSS2} & for \quad r = TSS1 \\ -\frac{\zeta_1^{TFA} k'^{TFA}_{21}}{\pi f'^{TFA}_1}\dot{q}_{TFA1} - \frac{\zeta_2^{TFA} k'^{TFA}_{22}}{\pi f'^{TFA}_2}\dot{q}_{TfA2} & for \quad r = TFA2 \\ -\frac{\zeta_1^{TSS} k'^{TSS}_{21}}{\pi f'^{TSS}_1}\dot{q}_{TSS1} - \frac{\zeta_2^{TSS} k'^{TSS}_{22}}{\pi f'^{TSS}_2}\dot{q}_{TSS2} & for \quad r = TSS2 \\ 0 & otherwise \end{cases}$$

where ζ_i^{TFA} and ζ_i^{TSS} represent the structural damping ratio of the tower for the i^{th} mode in the fore-aft and side-to-side directions, $TwrFADmp(i)/100$ and $TwrSSDmp(i)/100$ respectively. Also, f'^{TFA}_i and f'^{TSS}_i represent the natural frequency of the tower for the i^{th} mode in the fore-aft and side-to-side directions respectively without tower-top mass or gravitational destiffening effects. That is,

$$f'^{TFA}_i = \frac{1}{2\pi}\sqrt{\frac{k'^{TFA}_{ii}}{m'^{TFA}_{ii}}} \quad \text{and} \quad f'^{TSS}_i = \frac{1}{2\pi}\sqrt{\frac{k'^{TSS}_{ii}}{m'^{TSS}_{ii}}}$$

where m'^{TFA}_{ii} and m'^{TSS}_{ii} represent the generalized mass of the tower for the i^{th} mode in the fore-aft and side-to-side directions respectively without tower-top mass effects as follows:

$$m'^{TFA}_{ij} = \int_0^{TwrFlexL} \mu^T(h)\phi_i^{TFA}(h)\phi_j^{TFA}(h)\,dh \quad (i,j=1,2)$$

and

$$m'^{TSS}_{ij} = \int_0^{TwrFlexL} \mu^T(h)\phi_i^{TSS}(h)\phi_j^{TSS}(h)\,dh \quad (i,j=1,2)$$

Thus,

$$\left[C\left(q,t\right)\right]\Big|_{ElasticT}=0$$

and

$$\left[C\left(q,t\right)\right]\Big|_{DampT}=0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{ElasticT}=\left\{\begin{matrix} \\ \\ \\ \\ \\ \\ -k'^{TFA}_{11}\,q_{TFA1}-k'^{TFA}_{12}\,q_{TFA2} \\ -k'^{TSS}_{11}\,q_{TSS1}-k'^{TSS}_{12}\,q_{TSS2} \\ -k'^{TFA}_{21}\,q_{TFA1}-k'^{TFA}_{22}\,q_{TFA2} \\ -k'^{TSS}_{21}\,q_{TSS1}-k'^{TSS}_{22}\,q_{TSS2} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{matrix}\right\}$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{DampT}=\left\{\begin{matrix} \\ \\ \\ \\ \\ \\ -\dfrac{\zeta^{TFA}_{1}k'^{TFA}_{11}}{\pi f'^{TFA}_{1}}\dot{q}_{TFA1}-\dfrac{\zeta^{TFA}_{2}k'^{TFA}_{12}}{\pi f'^{TFA}_{2}}\dot{q}_{TFA2} \\ -\dfrac{\zeta^{TSS}_{1}k'^{TSS}_{11}}{\pi f'^{TSS}_{1}}\dot{q}_{TSS1}-\dfrac{\zeta^{TSS}_{2}k'^{TSS}_{12}}{\pi f'^{TSS}_{2}}\dot{q}_{TSS2} \\ -\dfrac{\zeta^{TFA}_{1}k'^{TFA}_{21}}{\pi f'^{TFA}_{1}}\dot{q}_{TFA1}-\dfrac{\zeta^{TFA}_{2}k'^{TFA}_{22}}{\pi f'^{TFA}_{2}}\dot{q}_{TFA2} \\ -\dfrac{\zeta^{TSS}_{1}k'^{TSS}_{21}}{\pi f'^{TSS}_{1}}\dot{q}_{TSS1}-\dfrac{\zeta^{TSS}_{2}k'^{TSS}_{22}}{\pi f'^{TSS}_{2}}\dot{q}_{TSS2} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{matrix}\right\}$$

$$F_r\Big|_{GravT} = \int_0^{TwrFlexL} {}^E\boldsymbol{v}_r^T(h)\cdot\left[-\mu^T(h)g\boldsymbol{z}_2\right]dh + {}^E\boldsymbol{v}_r^O\cdot\left(-YawBrMass\cdot g\boldsymbol{z}_2\right) \quad (r=3,4,\ldots,10)$$

Thus,

$$\left[C(q,t)\right]\Big|_{GravT} = 0$$

$$\left\{-f(\dot{q},q,t)\right\}\Big|_{GravT}(Row) = -\int_0^{TwrFlexL} \mu^T(h)g\,{}^E\boldsymbol{v}_{Row}^T(h)\cdot\boldsymbol{z}_2dh - YawBrMass\cdot g\,{}^E\boldsymbol{v}_{Row}^O\cdot\boldsymbol{z}_2 \quad (Row=3,4,\ldots,10)$$

$$F_r\Big|_{AeroT} = \int_0^{TwrFlexL} \left[{}^E\boldsymbol{v}_r^T(h)\cdot\boldsymbol{F}_{AeroT}^T(h) + {}^E\boldsymbol{\omega}_r^F(h)\cdot\boldsymbol{M}_{AeroT}^F(h)\right]dh \quad (r=1,2,\ldots,10)$$

where $\boldsymbol{F}_{AeroT}^T(h)$ and $\boldsymbol{M}_{AeroT}^F(h)$ are aerodynamic forces and moments applied to point T on the tower respectively *expressed per unit height*.

Thus,

$$\left[C(q,t)\right]\Big|_{AeroT} = 0$$

$$\left\{-f(\dot{q},q,t)\right\}\Big|_{AeroT}(Row) = \int_0^{TwrFlexL} \left[{}^E\boldsymbol{v}_{Row}^T(h)\cdot\boldsymbol{F}_{AeroT}^T(h) + {}^E\boldsymbol{\omega}_{Row}^F(h)\cdot\boldsymbol{M}_{AeroT}^F(h)\right]dh \quad (Row=1,2,\ldots,10)$$

<u>Yaw:</u>

The yaw spring and yaw damper bring about yaw moments.

$$F_r\Big|_{SpringYaw} = \begin{cases} -YawSpr\left(q_{Yaw} - YawNeut\right) & for \quad r = Yaw \\ 0 & otherwise \end{cases}$$

and

$$F_r\Big|_{DampYaw} = \begin{cases} -YawDamp \cdot \dot{q}_{Yaw} & for \quad r = Yaw \\ 0 & otherwise \end{cases}$$

Thus,

$$\left[C(q,t)\right]_{SpringYaw} = 0$$

and

$$\left[C(q,t)\right]_{DampYaw} = 0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{SpringYaw} = \begin{Bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ -YawSpr\left(q_{Yaw} - YawNeut\right) \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{Bmatrix}$$

and

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{DampYaw} = \begin{Bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ -YawDamp \cdot \dot{q}_{Yaw} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{Bmatrix}$$

Nacelle:

The rigid lump mass of the nacelle brings about generalized inertia forces and generalized active forces associated with nacelle weight.

$$F_r^*\big|_N = {}^E\boldsymbol{v}_r^U \cdot \left(-m^N\, {}^E\boldsymbol{a}^U\right) + {}^E\boldsymbol{\omega}_r^N \cdot \left(-{}^E\dot{\boldsymbol{H}}^N\right) \quad (r = 1,2,\ldots,22) \qquad \text{where} \qquad m^N = NacMass$$

Thus,

$$F_r^*\big|_N = {}^E\boldsymbol{v}_r^U \cdot \left(-m^N\, {}^E\boldsymbol{a}^U\right) + {}^E\boldsymbol{\omega}_r^N \cdot \left(-\overline{\overline{\boldsymbol{I}}}^N \cdot {}^E\boldsymbol{\alpha}^N - {}^E\boldsymbol{\omega}^N \times \overline{\overline{\boldsymbol{I}}}^N \cdot {}^E\boldsymbol{\omega}^N\right) \quad (r = 1,2,\ldots,22)$$

where $\overline{\overline{\boldsymbol{I}}}^N = \left[NacYIner - m^N\left(NacCMxn^2 + NacCMyn^2\right)\right]\boldsymbol{d_2 d_2}$

Or,

$$F_r^*\big|_N = {}^E\boldsymbol{v}_r^U \cdot \left(-m^N\left\{\left(\sum_{i=1}^{11} {}^E\boldsymbol{v}_i^U \ddot{q}_i\right) + \left[\sum_{i=4}^{11} \frac{d}{dt}\left({}^E\boldsymbol{v}_i^U\right)\dot{q}_i\right]\right\}\right) + {}^E\boldsymbol{\omega}_r^N \cdot \left(-\overline{\overline{\boldsymbol{I}}}^N \cdot \left\{\left(\sum_{i=4}^{11} {}^E\boldsymbol{\omega}_i^N \ddot{q}_i\right) + \left[\sum_{i=7}^{11} \frac{d}{dt}\left({}^E\boldsymbol{\omega}_i^N\right)\dot{q}_i\right]\right\} - {}^E\boldsymbol{\omega}^N \times \overline{\overline{\boldsymbol{I}}}^N \cdot {}^E\boldsymbol{\omega}^N\right) \quad (r = 1,2,\ldots,11)$$

Thus,

$$\left[C(q,t)\right]\big|_N (Row,Col) = m^N\, {}^E\boldsymbol{v}_{Row}^U \cdot {}^E\boldsymbol{v}_{Col}^U + {}^E\boldsymbol{\omega}_{Row}^N \cdot \overline{\overline{\boldsymbol{I}}}^N \cdot {}^E\boldsymbol{\omega}_{Col}^N \quad (Row,Col = 1,2,\ldots,11)$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\big|_N (Row) = -m^N\, {}^E\boldsymbol{v}_{Row}^U \cdot \left[\sum_{i=4}^{11} \frac{d}{dt}\left({}^E\boldsymbol{v}_i^U\right)\dot{q}_i\right] - {}^E\boldsymbol{\omega}_{Row}^N \cdot \left\{\overline{\overline{\boldsymbol{I}}}^N \cdot \left[\sum_{i=7}^{11} \frac{d}{dt}\left({}^E\boldsymbol{\omega}_i^N\right)\dot{q}_i\right] + {}^E\boldsymbol{\omega}^N \times \overline{\overline{\boldsymbol{I}}}^N \cdot {}^E\boldsymbol{\omega}^N\right\} \quad (Row = 1,2,\ldots,11)$$

$$F_r\big|_{GravN} = {}^E\boldsymbol{v}_r^U \cdot \left(-m^N g \boldsymbol{z_2}\right) \quad (r = 3,4,\ldots,11)$$

Thus,

$$\left[C(q,t)\right]\big|_{GravN} = 0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\big|_{GravN} (Row) = -m^N g\, {}^E\boldsymbol{v}_{Row}^U \cdot \boldsymbol{z_2} \quad (Row = 3,4,\ldots,11)$$

Rotor-Furl:

The rotor-furl springs (linear and stops) and rotor-furl dampers (Coulomb, linear, and stops) bring about rotor-furl moments.

$$F_r\Big|_{SpringRF} = \begin{cases} -RFrlSpr \cdot q_{RFrl} \\ \quad - IF\left[q_{RFrl} > RFrlUSSP, RFrlUSSpr\left(q_{RFrl} - RFrlUSSP\right), 0\right] & for \quad r = RFrl \\ \quad - IF\left[q_{RFrl} < RFrlDSSP, RFrlDSSpr\left(q_{RFrl} - RFrlDSSP\right), 0\right] \\ 0 & otherwise \end{cases}$$

and

$$F_r\Big|_{DampRF} = \begin{cases} -RFrlDmp \cdot \dot{q}_{RFrl} - IF\left[\dot{q}_{RFrl} <> 0, RFrlCDmp \cdot SIGN\left(\dot{q}_{RFrl}\right), 0\right] \\ \quad - IF\left[q_{RFrl} > RFrlUSDP, RFrlUSDmp \cdot \dot{q}_{RFrl}, 0\right] & for \quad r = RFrl \\ \quad - IF\left[q_{RFrl} < RFrlDSDP, RFrlDSDmp \cdot \dot{q}_{RFrl}, 0\right] \\ 0 & otherwise \end{cases}$$

Thus,

$$\left[C\left(q,t\right)\right]\Big|_{SpringRF}=0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{SpringRF}=\left\{\begin{array}{l}\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\-RFrlSpr\cdot q_{RFrl}\\\quad-IF\left[q_{RFrl}>RFrlUSSP,RFrlUSSpr\left(q_{RFrl}-RFrlUSSP\right),0\right]\\\quad-IF\left[q_{RFrl}<RFrlDSSP,RFrlDSSpr\left(q_{RFrl}-RFrlDSSP\right),0\right]\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\end{array}\right\}$$

and

$$\left[C\left(q,t \right) \right]\Big|_{DampRF} = 0$$

$$\left\{ -f\left(\dot{q},q,t \right) \right\}\Big|_{DampRF} = \left\{ \begin{array}{l} \vdots \\ -RFrlDmp \cdot \dot{q}_{RFrl} - IF\left[\dot{q}_{RFrl} <> 0, RFrlCDmp \cdot SIGN\left(\dot{q}_{RFrl} \right), 0 \right] \\ \quad - IF\left[q_{RFrl} > RFrlUSDP, RFrlUSDmp \cdot \dot{q}_{RFrl}, 0 \right] \\ \quad - IF\left[q_{RFrl} < RFrlDSDP, RFrlDSDmp \cdot \dot{q}_{RFrl}, 0 \right] \\ \vdots \end{array} \right\}$$

Structure That Furls with the Rotor (Not Including Rotor):

The rigid lump mass of the structure that furls with the rotor (not including the rotor) brings about generalized inertia forces and generalized active forces associated with the structure's weight.

$$F_r^*\Big|_R = {}^E\boldsymbol{v}_r^D \cdot \left(-m^R\,{}^E\boldsymbol{a}^D\right) + {}^E\boldsymbol{\omega}_r^R \cdot \left(-{}^E\dot{\boldsymbol{H}}^R\right) \quad (r = 1,2,\ldots,22) \qquad \text{where} \qquad m^R = RFrlMass$$

Thus,

$$F_r^*\Big|_R = {}^E\boldsymbol{v}_r^D \cdot \left(-m^R\,{}^E\boldsymbol{a}^D\right) + {}^E\boldsymbol{\omega}_r^R \cdot \left(-\bar{\bar{\boldsymbol{I}}}^R \cdot {}^E\boldsymbol{\alpha}^R - {}^E\boldsymbol{\omega}^R \times \bar{\bar{\boldsymbol{I}}}^R \cdot {}^E\boldsymbol{\omega}^R\right) \quad (r = 1,2,\ldots,22)$$

where $\bar{\bar{\boldsymbol{I}}}^R = \left[RFrlIner - m^R\left|\boldsymbol{r}^{VD} - \boldsymbol{r}^{VD}\cdot\boldsymbol{rfa}\,\boldsymbol{rfa}\right|^2\right]\boldsymbol{rfa}\,\boldsymbol{rfa}$ or

$$\bar{\bar{\boldsymbol{I}}}^R = \left(RFrlIner - RFrlMass\left\{ \begin{array}{l} \left(RFrlCMxn - RFrlPntxn\right)^2\left[1-\cos^2\left(RFrlSkew\right)\cos^2\left(RFrlTilt\right)\right] \\ \quad +\left(RFrlCMzn - RFrlPntzn\right)^2\cos^2\left(RFrlTilt\right) \\ \quad +\left(RFrlCMyn - RFrlPntyn\right)^2\left[1-\sin^2\left(RFrlSkew\right)\cos^2\left(RFrlTilt\right)\right] \\ \quad -2\left[\begin{array}{l} \left(RFrlCMxn - RFrlPntxn\right)\left(RFrlCMzn - RFrlPntzn\right)\cos\left(RFrlSkew\right)\cos\left(RFrlTilt\right)\sin\left(RFrlTilt\right) \\ \quad +\left(RFrlCMxn - RFrlPntxn\right)\left(RFrlCMyn - RFrlPntyn\right)\cos\left(RFrlSkew\right)\sin\left(RFrlSkew\right)\cos^2\left(RFrlTilt\right) \\ \quad +\left(RFrlCMzn - RFrlPntzn\right)\left(RFrlCMyn - RFrlPntyn\right)\sin\left(RFrlSkew\right)\cos\left(RFrlTilt\right)\sin\left(RFrlTilt\right) \end{array}\right] \end{array}\right\}\right)\boldsymbol{rfa}\,\boldsymbol{rfa}$$

Or,

$$F_r^*\Big|_R = {}^E\boldsymbol{v}_r^D \cdot \left(-m^R\left\{\left(\sum_{i=1}^{12} {}^E\boldsymbol{v}_i^D\ddot{q}_i\right) + \left[\sum_{i=4}^{12}\frac{d}{dt}\left({}^E\boldsymbol{v}_i^D\right)\dot{q}_i\right]\right\}\right) + {}^E\boldsymbol{\omega}_r^R \cdot \left(-\bar{\bar{\boldsymbol{I}}}^R \cdot \left\{\left(\sum_{i=4}^{12} {}^E\boldsymbol{\omega}_i^R\ddot{q}_i\right) + \left[\sum_{i=7}^{12}\frac{d}{dt}\left({}^E\boldsymbol{\omega}_i^R\right)\dot{q}_i\right]\right\} - {}^E\boldsymbol{\omega}^R \times \bar{\bar{\boldsymbol{I}}}^R \cdot {}^E\boldsymbol{\omega}^R\right) \quad (r = 1,2,\ldots,12)$$

Thus,

$$\left[C(q,t)\right]\Big|_R (Row,Col) = m^R\,{}^E\boldsymbol{v}_{Row}^D \cdot {}^E\boldsymbol{v}_{Col}^D + {}^E\boldsymbol{\omega}_{Row}^R \cdot \bar{\bar{\boldsymbol{I}}}^R \cdot {}^E\boldsymbol{\omega}_{Col}^R \quad (Row,Col = 1,2,\ldots,12)$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_R (Row) = -m^R\,{}^E\boldsymbol{v}_{Row}^D \cdot \left[\sum_{i=4}^{12}\frac{d}{dt}\left({}^E\boldsymbol{v}_i^D\right)\dot{q}_i\right] - {}^E\boldsymbol{\omega}_{Row}^R \cdot \left\{\bar{\bar{\boldsymbol{I}}}^R \cdot \left[\sum_{i=7}^{12}\frac{d}{dt}\left({}^E\boldsymbol{\omega}_i^R\right)\dot{q}_i\right] + {}^E\boldsymbol{\omega}^R \times \bar{\bar{\boldsymbol{I}}}^R \cdot {}^E\boldsymbol{\omega}^R\right\} \quad (Row = 1,2,\ldots,12)$$

$$F_r\Big|_{GravR} = {}^E\boldsymbol{v}_r^{\boldsymbol{D}} \cdot \left(-m^R g \boldsymbol{z}_2\right) \quad \left(r = 3,4,\ldots,12\right)$$

Thus,

$$\left[C\left(q,t\right)\right]\Big|_{GravR} = 0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{GravR}\left(Row\right) = -m^R g \, {}^E\boldsymbol{v}_{\boldsymbol{Row}}^{\boldsymbol{D}} \cdot \boldsymbol{z}_2 \quad \left(Row = 3,4,\ldots,12\right)$$

<u>Teeter:</u>
The teeter springs (soft and hard stop) and teeter dampers (Coulomb and soft stop) bring about teeter moments.

$$F_r\Big|_{SpringTeet} = \begin{cases} -IF\left[\left|q_{Teet}\right| > TeetSStP, TeetSSSp \cdot SIGN\left(q_{Teet}\right)\left(\left|q_{Teet}\right| - TeetSStP\right), 0\right] & \\ \quad -IF\left[\left|q_{Teet}\right| > TeetHStP, TeetHSSp \cdot SIGN\left(q_{Teet}\right)\left(\left|q_{Teet}\right| - TeetHStP\right), 0\right] & for \quad r = Teet \\ 0 & otherwise \end{cases}$$

and

$$F_r\Big|_{DampTeet} = \begin{cases} -IF\left[\dot{q}_{Teet} <> 0, TeetCDmp \cdot SIGN\left(\dot{q}_{Teet}\right), 0\right] & \\ \quad -IF\left[\left|q_{Teet}\right| > TeetDmpP, TeetDmp \cdot \dot{q}_{Teet}, 0\right] & for \quad r = Teet \\ 0 & otherwise \end{cases}$$

Thus,

$$\left[C(q,t)\right]\Big|_{SpringTeet} = 0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{SpringTeet} = \left\{\begin{array}{l} \vdots \\ -IF\left[\left|q_{Teet}\right| > TeetSStP, TeetSSSp \cdot SIGN\left(q_{Teet}\right)\left(\left|q_{Teet}\right| - TeetSStP\right), 0\right] \\ \quad - IF\left[\left|q_{Teet}\right| > TeetHStP, TeetHSSp \cdot SIGN\left(q_{Teet}\right)\left(\left|q_{Teet}\right| - TeetHStP\right), 0\right] \end{array}\right\}$$

and

$$\left[C\left(q,t\right)\right]_{DampTeet}=0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{DampTeet}=\left\{\begin{matrix}\cdots\\ -IF\left[\dot{q}_{Teet}<>0,TeetCDmp\cdot SIGN\left(\dot{q}_{Teet}\right),0\right]\\ -IF\left[\left|q_{Teet}\right|>TeetDmpP,TeetDmp\cdot\dot{q}_{Teet},0\right]\end{matrix}\right\}$$

Hub:
The rigid lump mass of the hub brings about generalized inertia forces and generalized active forces associated with hub weight.

$$F_r^*\Big|_H = {}^E\boldsymbol{v}_r^C \cdot \left(-m^H\, {}^E\boldsymbol{a}^C\right) + {}^E\boldsymbol{\omega}_r^H \cdot \left(-{}^E\dot{\boldsymbol{H}}^H\right) \quad (r = 1,2,\ldots,22) \qquad \text{where} \qquad m^H = HubMass$$

Thus,

$$F_r^*\Big|_H = {}^E\boldsymbol{v}_r^C \cdot \left(-m^H\, {}^E\boldsymbol{a}^C\right) + {}^E\boldsymbol{\omega}_r^H \cdot \left(-\overline{\overline{\boldsymbol{I}}}^H \cdot {}^E\boldsymbol{\alpha}^H - {}^E\boldsymbol{\omega}^H \times \overline{\overline{\boldsymbol{I}}}^H \cdot {}^E\boldsymbol{\omega}^H\right) \quad (r = 1,2,\ldots,22)$$

where
$$\overline{\overline{\boldsymbol{I}}}^N = \left[\frac{HubIner - m^H\left(UndSling - HubCM\right)^2}{cos^2\left(Delta3\right)}\right]\boldsymbol{g_1 g_1} + \left[\frac{HubIner - m^H\left(UndSling - HubCM\right)^2}{cos^2\left(Delta3\right)}\right]\boldsymbol{g_2 g_2}$$

since it is assumed that the hub is essentially a uniform rod directed along the $\boldsymbol{g_3}$ axis and passing through the hub center of mass location (point C).

Note that if: $\left[\frac{HubIner - m^H\left(UndSling - HubCM\right)^2}{cos^2\left(Delta3\right)}\right] < 0$, then there must be an error in the input file.

Or,

$$\begin{aligned} F_r^*\Big|_H = {}^E\boldsymbol{v}_r^C \cdot \Bigg(-m^H \Bigg\{ \left(\sum_{i=1}^{14} {}^E\boldsymbol{v}_i^C \ddot{q}_i\right) + {}^E\boldsymbol{v}_{Teet}^C \ddot{q}_{Teet} + \left[\sum_{i=4}^{14} \frac{d}{dt}\left({}^E\boldsymbol{v}_i^C\right)\dot{q}_i\right] + \frac{d}{dt}\left({}^E\boldsymbol{v}_{Teet}^C\right)\dot{q}_{Teet} \Bigg\}\Bigg) \\ + {}^E\boldsymbol{\omega}_r^H \cdot \Bigg(-\overline{\overline{\boldsymbol{I}}}^H \cdot \Bigg\{ \left(\sum_{i=4}^{14} {}^E\boldsymbol{\omega}_i^H \ddot{q}_i\right) + {}^E\boldsymbol{\omega}_{Teet}^H \ddot{q}_{Teet} + \left[\sum_{i=7}^{14} \frac{d}{dt}\left({}^E\boldsymbol{\omega}_i^H\right)\dot{q}_i\right] + \frac{d}{dt}\left({}^E\boldsymbol{\omega}_{Teet}^H\right)\dot{q}_{Teet} \Bigg\} - {}^E\boldsymbol{\omega}^H \times \overline{\overline{\boldsymbol{I}}}^H \cdot {}^E\boldsymbol{\omega}^H \Bigg) \end{aligned} \quad (r = 1,2,\ldots,14;Teet)$$

Thus,

$$\left[C(q,t)\right]\Big|_H (Row,Col) = m^H\, {}^E\boldsymbol{v}_{Row}^C \cdot {}^E\boldsymbol{v}_{Col}^C + {}^E\boldsymbol{\omega}_{Row}^H \cdot \overline{\overline{\boldsymbol{I}}}^H \cdot {}^E\boldsymbol{\omega}_{Col}^H \quad (Row,Col = 1,2,\ldots,14;22)$$

$$\begin{aligned} \left\{-f\left(\dot{q},q,t\right)\right\}\Big|_H (Row) = -m^H\, {}^E\boldsymbol{v}_{Row}^C \cdot \left\{\left[\sum_{i=4}^{14} \frac{d}{dt}\left({}^E\boldsymbol{v}_i^C\right)\dot{q}_i\right] + \frac{d}{dt}\left({}^E\boldsymbol{v}_{Teet}^C\right)\dot{q}_{Teet}\right\} \\ - {}^E\boldsymbol{\omega}_{Row}^H \cdot \left(\overline{\overline{\boldsymbol{I}}}^H \cdot \left\{\left[\sum_{i=7}^{14} \frac{d}{dt}\left({}^E\boldsymbol{\omega}_i^H\right)\dot{q}_i\right] + \frac{d}{dt}\left({}^E\boldsymbol{\omega}_{Teet}^H\right)\dot{q}_{Teet}\right\} + {}^E\boldsymbol{\omega}^H \times \overline{\overline{\boldsymbol{I}}}^H \cdot {}^E\boldsymbol{\omega}^H\right) \end{aligned} \quad (Row = 1,2,\ldots,14;22)$$

$$F_r\big|_{GravH} = {}^{E}\boldsymbol{v}_r^{C} \cdot \left(-m^H g \boldsymbol{z_2}\right) \quad \left(r = 3,4,\ldots,14;Teet\right)$$

Thus,

$$\left[C(q,t)\right]_{GravH} = 0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{GravH}\left(Row\right) = -m^H g\, {}^{E}\boldsymbol{v}_{Row}^{C} \cdot \boldsymbol{z_2} \quad \left(Row = 3,4,\ldots,14;22\right)$$

Blade 1:

The distributed properties of blade 1 bring about generalized inertia forces and generalized active forces associated with blade elasticity, blade damping, blade weight, and blade aerodynamics.

$$\left. F_r^* \right|_{B1} = -\int_0^{BldFlexL} \mu^{B1}(r)\, {}^E\boldsymbol{v}_r^{S1}(r) \cdot {}^E\boldsymbol{a}^{S1}(r)\, dr - m^{B1Tip}\, {}^E\boldsymbol{v}_r^{S1}(BldFlexL) \cdot {}^E\boldsymbol{a}^{S1}(BldFlexL) \quad (r = 1, 2, \ldots, 22)$$

where $\mu^{B1}(r) = AdjBlMs^{B1} \cdot BMassDen^{B1}(r)$ and $m^{B1Tip} = TipMass(1)$

Or,

$$\begin{aligned} \left. F_r^* \right|_{B1} = &-\int_0^{BldFlexL} \mu^{B1}(r)\, {}^E\boldsymbol{v}_r^{S1}(r) \cdot \left\{ \begin{aligned} &\left(\sum_{i=1}^{14} {}^E\boldsymbol{v}_i^{S1}(r)\, \ddot{q}_i \right) + \left(\sum_{i=16}^{18} {}^E\boldsymbol{v}_i^{S1}(r)\, \ddot{q}_i \right) + {}^E\boldsymbol{v}_{Teet}^{S1}(r)\, \ddot{q}_{Teet} \\ &+ \left[\sum_{i=4}^{14} \frac{d}{dt}\left({}^E\boldsymbol{v}_i^{S1}(r) \right) \dot{q}_i \right] + \left[\sum_{i=16}^{18} \frac{d}{dt}\left({}^E\boldsymbol{v}_i^{S1}(r) \right) \dot{q}_i \right] + \frac{d}{dt}\left({}^E\boldsymbol{v}_{Teet}^{S1}(r) \right) \dot{q}_{Teet} \end{aligned} \right\} dr \\ &- m^{B1Tip}\, {}^E\boldsymbol{v}_r^{S1}(BldFlexL) \cdot \left\{ \begin{aligned} &\left(\sum_{i=1}^{14} {}^E\boldsymbol{v}_i^{S1}(BldFlexL)\, \ddot{q}_i \right) + \left(\sum_{i=16}^{18} {}^E\boldsymbol{v}_i^{S1}(BldFlexL)\, \ddot{q}_i \right) + {}^E\boldsymbol{v}_{Teet}^{S1}(BldFlexL)\, \ddot{q}_{Teet} \\ &+ \left[\sum_{i=4}^{14} \frac{d}{dt}\left({}^E\boldsymbol{v}_i^{S1}(BldFlexL) \right) \dot{q}_i \right] + \left[\sum_{i=16}^{18} \frac{d}{dt}\left({}^E\boldsymbol{v}_i^{S1}(BldFlexL) \right) \dot{q}_i \right] \\ &+ \frac{d}{dt}\left({}^E\boldsymbol{v}_{Teet}^{S1}(BldFlexL) \right) \dot{q}_{Teet} \end{aligned} \right\} \quad (r = 1, 2, \ldots, 14; 16, 17, 18; Teet) \end{aligned}$$

Thus,

$$\left[C(q,t)\right]\Big|_{B1}(Row,Col)=\int_0^{BldFlexL}\mu^{B1}(r)\,{}^{E}\boldsymbol{v}^{S1}_{Row}(r)\cdot{}^{E}\boldsymbol{v}^{S1}_{Col}(r)\,dr+m^{B1Tip}\,{}^{E}\boldsymbol{v}^{S1}_{Row}(BldFlexL)\cdot{}^{E}\boldsymbol{v}^{S1}_{Col}(BldFlexL)\quad(Row,Col=1,2,\ldots,14;16,17,18;22)$$

$$\begin{aligned}\left\{-f(\dot{q},q,t)\right\}\Big|_{B1}(Row)=&-\int_0^{BldFlexL}\mu^{B1}(r)\,{}^{E}\boldsymbol{v}^{S1}_{Row}(r)\cdot\left\{\left[\sum_{i=4}^{14}\frac{d}{dt}\left({}^{E}\boldsymbol{v}^{S1}_{i}(r)\right)\dot{q}_i\right]+\left[\sum_{i=16}^{18}\frac{d}{dt}\left({}^{E}\boldsymbol{v}^{S1}_{i}(r)\right)\dot{q}_i\right]+\frac{d}{dt}\left({}^{E}\boldsymbol{v}^{S1}_{Teet}(r)\right)\dot{q}_{Teet}\right\}dr\\&-m^{B1Tip}\,{}^{E}\boldsymbol{v}^{S1}_{Row}(BldFlexL)\cdot\left\{\begin{array}{l}\left[\sum_{i=4}^{14}\frac{d}{dt}\left({}^{E}\boldsymbol{v}^{S1}_{i}(BldFlexL)\right)\dot{q}_i\right]+\left[\sum_{i=16}^{18}\frac{d}{dt}\left({}^{E}\boldsymbol{v}^{S1}_{i}(BldFlexL)\right)\dot{q}_i\right]\\+\frac{d}{dt}\left({}^{E}\boldsymbol{v}^{S1}_{Teet}(BldFlexL)\right)\dot{q}_{Teet}\end{array}\right\}\quad(Row=1,2,\ldots,14;16,17,18;22)\end{aligned}$$

$$F_r\Big|_{ElasticB1}=-\frac{\partial V'^{B1}}{\partial q_r}\quad(r=1,2,\ldots,22)$$

So,

$$F_r\Big|_{ElasticB1}=\begin{cases}-k'^{B1F}_{11}q_{B1F1}-k'^{B1F}_{12}q_{B1F2} & for\quad r=B1F1\\-k'^{B1E}_{11}q_{B1E1} & for\quad r=B1E1\\-k'^{B1F}_{21}q_{B1F1}-k'^{B1F}_{22}q_{B1F2} & for\quad r=B1F2\\0 & otherwise\end{cases}$$

where k'^{B1F}_{ij} and k'^{B1E}_{11} are the generalized stiffnesses of blade 1 in the local flap and local edge directions respectively when centrifugal-stiffening effects are *not* included as follows:

$$k'^{B1F}_{ij}=\sqrt{FlStTunr^{B1}(i)\,FlStTunr^{B1}(j)}\int_0^{BldFlexL}EI^{B1F}(r)\frac{d^2\phi^{B1F}_i(r)}{dr^2}\frac{d^2\phi^{B1F}_j(r)}{dr^2}dr\quad(i,j=1,2)$$

where

$$EI^{B1F}(r)=AdjFlSt^{B1}\cdot FlpStff^{B1}(r)$$

and

$$k'^{B1E}_{11}=\int_0^{BldFlexL}EI^{B1E}(r)\left[\frac{d^2\phi^{B1E}_1(r)}{dr^2}\right]^2dr$$

where

$$EI^{B1E}(r)=AdjEdSt^{B1}\cdot EdgStff^{B1}(r)$$

Similarly, when using the Rayleigh damping technique where the damping is assumed proportional to the stiffness, then

$$F_r\Big|_{DampB1} = \begin{cases} -\dfrac{\zeta_1^{B1F} k{'}_{11}^{B1F}}{\pi f{'}_1^{B1F}} \dot{q}_{B1F1} - \dfrac{\zeta_2^{B1F} k{'}_{12}^{B1F}}{\pi f{'}_2^{B1F}} \dot{q}_{B1F2} & for \quad r = B1F1 \\ -\dfrac{\zeta_1^{B1E} k{'}_{11}^{B1E}}{\pi f{'}_1^{B1E}} \dot{q}_{B1E1} & for \quad r = B1E1 \\ -\dfrac{\zeta_1^{B1F} k{'}_{21}^{B1F}}{\pi f{'}_1^{B1F}} \dot{q}_{B1F1} - \dfrac{\zeta_2^{B1F} k{'}_{22}^{B1F}}{\pi f{'}_2^{B1F}} \dot{q}_{B1F2} & for \quad r = B1F2 \\ 0 & otherwise \end{cases}$$

where ζ_i^{B1F} and ζ_i^{B1E} represent the structural damping ratio of blade 1 for the i^{th} mode in the local flap and edge directions, $BldFlDmp^{B1}(i)/100$ and $BldEdDmp^{B1}(i)/100$ respectively. Also, $f{'}_i^{B1F}$ and $f{'}_i^{B1E}$ represent the natural frequency of blade 1 for the i^{th} mode in the local flap and edge directions respectively without centrifugal-stiffening effects. That is,

$$f{'}_i^{B1F} = \frac{1}{2\pi}\sqrt{\frac{k{'}_{ii}^{B1F}}{m{'}_{ii}^{B1F}}} \quad \text{and} \quad f{'}_i^{B1E} = \frac{1}{2\pi}\sqrt{\frac{k{'}_{ii}^{B1E}}{m{'}_{ii}^{B1E}}}$$

where $m{'}_{ii}^{B1F}$ and $m{'}_{ii}^{B1E}$ represent the generalized mass of blade 1 for the i^{th} mode in the local flap and edge directions respectively without centrifugal-stiffening and tip mass effects as follows:

$$m{'}_{ij}^{B1F} = \int_0^{BldFlexL} \mu^{B1}(r)\phi_i^{B1F}(r)\phi_j^{B1F}(r)\,dr \quad (i, j = 1,2)$$

and

$$m{'}_{11}^{B1E} = \int_0^{BldFlexL} \mu^{B1}(r)\left[\phi_1^{B1E}(r)\right]^2 dr$$

Thus,

$$\left[C\left(q,t\right)\right]_{ElasticB1}=0 \qquad \text{and} \qquad \left[C\left(q,t\right)\right]_{DampB1}=0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{ElasticB1}=\begin{Bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ -k'^{B1F}_{11}\,q_{B1F1}-k'^{B1F}_{12}\,q_{B1F2} \\ -k'^{B1E}_{11}\,q_{B1E1} \\ -k'^{B1F}_{21}\,q_{B1F1}-k'^{B1F}_{22}\,q_{B1F2} \\ \\ \\ \\ \\ \end{Bmatrix}$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{DampB1}=\begin{Bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ -\dfrac{\zeta^{B1F}_{1}k'^{B1F}_{11}}{\pi f'^{B1F}_{1}}\dot{q}_{B1F1}-\dfrac{\zeta^{B1F}_{2}k'^{B1F}_{12}}{\pi f'^{B1F}_{2}}\dot{q}_{B1F2} \\ -\dfrac{\zeta^{B1E}_{1}k'^{B1E}_{11}}{\pi f'^{B1E}_{1}}\dot{q}_{B1E1} \\ -\dfrac{\zeta^{B1F}_{1}k'^{B1F}_{21}}{\pi f'^{B1F}_{1}}\dot{q}_{B1F1}-\dfrac{\zeta^{B1F}_{2}k'^{B1F}_{22}}{\pi f'^{B1F}_{2}}\dot{q}_{B1F2} \\ \\ \\ \\ \\ \end{Bmatrix}$$

$$F_r\big|_{GravB1} = -\int_0^{BldFlexL} \mu^{B1}(r)\, g\, {}^E\boldsymbol{v}_r^{S1}(r)\cdot \boldsymbol{z}_2 dr - m^{B1Tip} g\, {}^E\boldsymbol{v}_r^{S1}(BldFlexL)\cdot \boldsymbol{z}_2 \quad (r = 3,4,\ldots,14;16,17,18;Teet)$$

Thus,

$$[C(q,t)]_{GravB1} = 0$$

$$\{-f(\dot{q},q,t)\}\big|_{GravB1}(Row) = -\int_0^{BldFlexL} \mu^{B1}(r)\, g\, {}^E\boldsymbol{v}_{Row}^{S1}(r)\cdot \boldsymbol{z}_2 dr - m^{B1Tip} g\, {}^E\boldsymbol{v}_{Row}^{S1}(BldFlexL)\cdot \boldsymbol{z}_2 \quad (Row = 3,4,\ldots,14;16,17,18;22)$$

$$F_r\big|_{AeroB1} = \int_0^{BldFlexL} \left[{}^E\boldsymbol{v}_r^{S1}(r)\cdot \boldsymbol{F}_{AeroB1}^{S1}(r) + {}^E\boldsymbol{\omega}_r^{M1}(r)\cdot \boldsymbol{M}_{AeroB1}^{M1}(r)\right] dr + {}^E\boldsymbol{v}_r^{S1}(BldFlexL)\cdot \boldsymbol{F}_{TipDragB1}^{S1}(BldFlexL) \quad (r = 1,2,\ldots,14;16,17,18;Teet)$$

where $\boldsymbol{F}_{AeroB1}^{S1}(r)$ and $\boldsymbol{M}_{AeroB1}^{M1}(r)$ are aerodynamic forces and pitching moments applied to point S1 on blade 1 respectively *expressed per unit span*. Note that $\boldsymbol{M}_{AeroB1}^{M1}(r)$ can include effects of both direct aerodynamic pitching moments (i.e., Cm) and aerodynamic pitching moments caused by an aerodynamic offset (i.e., moments due to aerodynamic lift and drag forces acting at a distance away from the center of mass of the blade element along the aerodynamic chord).

Thus,

$$[C(q,t)]_{AeroB1} = 0$$

$$\begin{aligned}\{-f(\dot{q},q,t)\}\big|_{AeroB1}(Row) = &\int_0^{BldFlexL} \left[{}^E\boldsymbol{v}_{Row}^{S1}(r)\cdot \boldsymbol{F}_{AeroB1}^{S1}(r) + {}^E\boldsymbol{\omega}_{Row}^{M1}(r)\cdot \boldsymbol{M}_{AeroB1}^{M1}(r)\right] dr \\ &+ {}^E\boldsymbol{v}_{Row}^{S1}(BldFlexL)\cdot \boldsymbol{F}_{TipDragB1}^{S1}(BldFlexL)\end{aligned} \quad (Row = 1,2,\ldots,14;16,17,18;22)$$

Blade 2:

Just like blade 1, the distributed properties of blade 2 bring about generalized inertia forces and generalized active forces associated with blade elasticity, blade damping, blade weight, and blade aerodynamics. The equations for $F_r^*\big|_{B2}$, $F_r\big|_{ElasticB2}$, $F_r\big|_{DampB2}$, $F_r\big|_{GravB2}$, and $F_r\big|_{AeroB2}$ are similar to those of blade 1.

Drivetrain:

The inertia of the drivetrain brings about generalized inertia forces and the load in the generator, high-speed shaft brake, gearbox (friction forces resulting from nonzero $GBoxEff$) and the flexibility of the low speed shaft bring about generalized active forces. Note that all of these equations assume that the rotor is spinning about the positive $\boldsymbol{c_1}$ axis (they assume that the rotor can't be forced to rotate in the opposite direction). This model works for any gearbox arrangement (including no gearbox, single stage, or multi-stage) as long as the generator rotates about the shaft axis (it may not be skewed relative to the shaft, even though it may rotate in the opposite direction of the low-speed shaft due to the gearbox stages). If there is no gearbox, simply set $GBRatio = GBoxEff = GenDir = 1$ ($GBReverse$ = False).

The mechanical torque within the generator is applied to the high speed shaft and equally and oppositely to the structure that furls with the rotor as follows:

$$F_r\big|_{Gen} = \left({}^E\boldsymbol{\omega}_r^G - {}^E\boldsymbol{\omega}_r^R \right) \cdot \boldsymbol{M}_{Gen}^G \quad \left(r = 1, 2, \ldots, 22 \right)$$

Thus,

$$F_r\big|_{Gen} = \begin{cases} {}^E\boldsymbol{\omega}_{GeAz}^G \cdot \boldsymbol{M}_{Gen}^G & for \quad r = GeAz \\ 0 & otherwise \end{cases} \qquad \text{where} \qquad \boldsymbol{M}_{Gen}^G = -GenDir \cdot T^{Gen}\left(GBRatio \cdot \dot{q}_{GeAz}, t \right) \boldsymbol{c_1}$$

Note that a positive T^{Gen} represents a load (positive power extracted) whereas a negative T^{Gen} represents a motoring-up situation (negative power extracted, or power input). Thus,

$$F_r\big|_{Gen} = \begin{cases} \left(GenDir \cdot GBRatio \cdot \boldsymbol{c_1} \right) \cdot \left[-GenDir \cdot T^{Gen}\left(GBRatio \cdot \dot{q}_{GeAz}, t \right) \boldsymbol{c_1} \right] & for \quad r = GeAz \\ 0 & otherwise \end{cases}$$

Or since $GenDir^2 = 1$,

$$F_r\big|_{Gen} = \begin{cases} -GBRatio \cdot T^{Gen}\left(GBRatio \cdot \dot{q}_{GeAz}, t \right) & for \quad r = GeAz \\ 0 & otherwise \end{cases}$$

Thus,

$$\left[C(q,t)\right]_{Gen}=0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{Gen}=\begin{Bmatrix}\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\-GBRatio\cdot T^{Gen}\left(GBRatio\cdot\dot{q}_{GeAz},t\right)\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\\\cdots\end{Bmatrix}$$

Similarly, the mechanical torque applied to the high-speed shaft from the high-speed shaft brake is applied equally and oppositely to the structure that furls with the rotor. Thus,

$$F_r\Big|_{Brake} = \begin{cases} {}^{E}\boldsymbol{\omega}^{G}_{\boldsymbol{GeAz}} \cdot \boldsymbol{M}^{G}_{\boldsymbol{Brake}} & for \quad r = GeAz \\ 0 & otherwise \end{cases}$$ where $$\boldsymbol{M}^{G}_{\boldsymbol{Brake}} = -GenDir \cdot T^{Brake}(t)\,\boldsymbol{c_1}$$ and where $$T^{Brake}(t) = HSSBrkT(t)$$

which is assumed to be positive in value always. Thus,

$$F_r\Big|_{Brake} = \begin{cases} -GBRatio \cdot T^{Brake}(t) & for \quad r = GeAz \\ 0 & otherwise \end{cases}$$

Thus,

$$\left[C\left(q,t\right)\right]\Big|_{Brake} = 0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{Brake} = \left\{\begin{array}{c} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ -GBRatio \cdot T^{Brake}\left(t\right) \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{array}\right\}$$

If the translational inertia of the drivetrain is assumed to be incorporated into that of the structure that furls with the rotor, then the high-speed shaft generator inertia generalized force is as follows:

$$F_r^*\Big|_G = {}^E\boldsymbol{\omega}_r^G \cdot \left(-\overline{\overline{\boldsymbol{I}}}^G \cdot {}^E\boldsymbol{\alpha}^G - {}^E\boldsymbol{\omega}^G \times \overline{\overline{\boldsymbol{I}}}^G \cdot {}^E\boldsymbol{\omega}^G \right) \quad (r = 1,2,\ldots,22) \qquad \text{where} \qquad \overline{\overline{\boldsymbol{I}}}^G = GenIner\boldsymbol{c}_1\boldsymbol{c}_1$$

or,

$$F_r^*\Big|_G = {}^E\boldsymbol{\omega}_r^G \cdot \left\{ -\overline{\overline{\boldsymbol{I}}}^G \cdot \left\{ \left(\sum_{i=4}^{13} {}^E\boldsymbol{\omega}_i^G \ddot{q}_i \right) + \left[\sum_{i=7}^{13} \frac{d}{dt}\left({}^E\boldsymbol{\omega}_i^G \right) \dot{q}_i \right] \right\} - {}^E\boldsymbol{\omega}^G \times \overline{\overline{\boldsymbol{I}}}^G \cdot {}^E\boldsymbol{\omega}^G \right\} \quad (r = 1,2,\ldots,22)$$

However, since $\boldsymbol{c}_1 \cdot \frac{d}{dt}\left({}^E\boldsymbol{\omega}_{GeAz}^G \right) \propto \boldsymbol{c}_1 \cdot \left({}^E\boldsymbol{\omega}^R \times \boldsymbol{c}_1 \right) = {}^E\boldsymbol{\omega}^R \cdot \left(\boldsymbol{c}_1 \times \boldsymbol{c}_1 \right) = 0$ (the first $\boldsymbol{c}_1$ coming from $\overline{\overline{\boldsymbol{I}}}^G$), this simplifies as follows:

$$F_r^*\Big|_G = {}^E\boldsymbol{\omega}_r^G \cdot \left\{ -\overline{\overline{\boldsymbol{I}}}^G \cdot \left\{ \left(\sum_{i=4}^{13} {}^E\boldsymbol{\omega}_i^G \ddot{q}_i \right) + \left[\sum_{i=7}^{12} \frac{d}{dt}\left({}^E\boldsymbol{\omega}_i^G \right) \dot{q}_i \right] \right\} - {}^E\boldsymbol{\omega}^G \times \overline{\overline{\boldsymbol{I}}}^G \cdot {}^E\boldsymbol{\omega}^G \right\} \quad (r = 1,2,\ldots,22)$$

Or,

$$F_r^*\Big|_G = \begin{cases} -{}^E\boldsymbol{\omega}_r^R \cdot \overline{\overline{\boldsymbol{I}}}^G \cdot \left\{ \left(\sum_{i=4}^{13} {}^E\boldsymbol{\omega}_i^G \ddot{q}_i \right) + \left[\sum_{i=7}^{12} \frac{d}{dt}\left({}^E\boldsymbol{\omega}_i^R \right) \dot{q}_i \right] \right\} - {}^E\boldsymbol{\omega}_r^R \cdot \left({}^E\boldsymbol{\omega}^G \times \overline{\overline{\boldsymbol{I}}}^G \cdot {}^E\boldsymbol{\omega}^G \right) & for \quad r = 4,5,\ldots,12 \\ -GenDir \cdot GenIner \cdot GBRatio \left\{ \left(\sum_{i=4}^{12} {}^E\boldsymbol{\omega}_i^R \ddot{q}_i \right) + \left[\sum_{i=7}^{12} \frac{d}{dt}\left({}^E\boldsymbol{\omega}_i^R \right) \dot{q}_i \right] \right\} \cdot \boldsymbol{c}_1 - GenIner \cdot GBRatio^2 \cdot \ddot{q}_{GeAz} & \\ \quad - GenDir \cdot GBRatio \boldsymbol{c}_1 \cdot \left({}^E\boldsymbol{\omega}^G \times \overline{\overline{\boldsymbol{I}}}^G \cdot {}^E\boldsymbol{\omega}^G \right) & for \quad r = GeAz \\ 0 & otherwise \end{cases}$$

However since $\boldsymbol{c}_1 \cdot \left({}^E\boldsymbol{\omega}^G \times \boldsymbol{c}_1 \right) = {}^E\boldsymbol{\omega}^G \cdot \left(\boldsymbol{c}_1 \times \boldsymbol{c}_1 \right) = 0$ (the first $\boldsymbol{c}_1$ coming from $\overline{\overline{\boldsymbol{I}}}^G$), this simplifies again as follows:

$$F_r^*\Big|_G = \begin{cases} -{}^E\boldsymbol{\omega}_r^R \cdot \overline{\overline{\boldsymbol{I}}}^G \cdot \left\{ \left(\sum_{i=4}^{13} {}^E\boldsymbol{\omega}_i^G \ddot{q}_i \right) + \left[\sum_{i=7}^{12} \frac{d}{dt}\left({}^E\boldsymbol{\omega}_i^R \right) \dot{q}_i \right] \right\} - {}^E\boldsymbol{\omega}_r^R \cdot \left({}^E\boldsymbol{\omega}^G \times \overline{\overline{\boldsymbol{I}}}^G \cdot {}^E\boldsymbol{\omega}^G \right) & for \quad r = 4,5,\ldots,12 \\ -GenDir \cdot GenIner \cdot GBRatio \left\{ \left(\sum_{i=4}^{12} {}^E\boldsymbol{\omega}_i^R \ddot{q}_i \right) + \left[\sum_{i=7}^{12} \frac{d}{dt}\left({}^E\boldsymbol{\omega}_i^R \right) \dot{q}_i \right] \right\} \cdot \boldsymbol{c}_1 - GenIner \cdot GBRatio^2 \cdot \ddot{q}_{GeAz} & for \quad r = GeAz \\ 0 & otherwise \end{cases}$$

The terms associated with DOFs 4,5,…,12 represent the fact that the rate of change of angular momentum of the generator can be considered as an additional torque on the structure that furls with the rotor (i.e., in addition to the torques on the structure transmitted directly from the low-speed shaft).

Thus,

$$\left[C(q,t)\right]\Big|_G = \begin{cases} {}^E\boldsymbol{\omega}^G_{Row} \cdot \overline{\overline{\boldsymbol{I}}}^G \cdot {}^E\boldsymbol{\omega}^G_{Col} & for \quad (Row,Col = 4,5,\ldots,13) \\ 0 & otherwise \end{cases}$$

$$\left\{-f(\dot{q},q,t)\right\}\Big|_G = \begin{cases} -{}^E\boldsymbol{\omega}^G_{Row} \cdot \left\{ \overline{\overline{\boldsymbol{I}}}^G \cdot \left[\sum_{i=7}^{12} \frac{d}{dt}\left({}^E\boldsymbol{\omega}^R_i\right)\dot{q}_i \right] + {}^E\boldsymbol{\omega}^G \times \overline{\overline{\boldsymbol{I}}}^G \cdot {}^E\boldsymbol{\omega}^G \right\} & for \quad (Row = 4,5,\ldots,12) \\ -{}^E\boldsymbol{\omega}^G_{Row} \cdot \overline{\overline{\boldsymbol{I}}}^G \cdot \left[\sum_{i=7}^{12} \frac{d}{dt}\left({}^E\boldsymbol{\omega}^R_i\right)\dot{q}_i \right] & for \quad (Row = 13) \\ 0 & otherwise \end{cases}$$

$$F_r\Big|_{ElasticDrive} = -\frac{\partial V_{Drive}}{\partial q_r} \quad (r = 1,2,\ldots,22) \qquad \text{where} \qquad V_{Drive} = \frac{1}{2} DTTorSpr \cdot q^2_{DrTr}$$

So,

$$F_r\Big|_{ElasticDrive} = \begin{cases} -DTTorSpr \cdot q_{DrTr} & for \quad r = DrTr \\ 0 & otherwise \end{cases} \qquad \text{and likewise} \qquad F_r\Big|_{DampDrive} = \begin{cases} -DTTorDmp \cdot \dot{q}_{DrTr} & for \quad r = DrTr \\ 0 & otherwise \end{cases}$$

Thus,

$$\left[C(q,t)\right]_{ElasticDrive}=0$$

and likewise

$$\left[C(q,t)\right]_{DampDrive}=0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{ElasticDrive}=\left\{\begin{array}{c}\cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ -DTTorSpr\cdot q_{DrTr}\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\end{array}\right\}$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{DampDrive}=\left\{\begin{array}{c}\cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ -DTTorDmp\cdot \dot{q}_{DrTr}\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\\ \cdots\end{array}\right\}$$

Similar to the generator and HSS brake, the mechanical friction torque applied to the high speed shaft is applied equally and oppositely to the structure that furls with the rotor. Thus,

$$F_r\Big|_{GBFric} = \begin{cases} {}^E\boldsymbol{\omega}^G_{\mathbf{GeAz}} \cdot \boldsymbol{M}^G_{\mathbf{GBFric}} & for \quad r = GeAz \\ 0 & otherwise \end{cases} \qquad \text{where} \qquad \boldsymbol{M}^G_{\mathbf{GBFric}} = -\frac{T^{GBFric}\left(\ddot{q},\dot{q},q,t\right)\boldsymbol{c_1}}{GBRatio \cdot GenDir}$$

where, from a free-body diagram of the high and low-speed shafts, it is easily seen that the friction torque applied on the LSS upon the gearbox entrance, $T^{GBFric}\left(\ddot{q},\dot{q},q,t\right)$, is always positive in value and equal to:

$$T^{GBFric}\left(\ddot{q},\dot{q},q,t\right) = \begin{bmatrix} GenIner \cdot GBRatio^2 \cdot \ddot{q}_{GeAz} + GenDir \cdot GenIner \cdot GBRatio\, {}^E\boldsymbol{\alpha}^R \cdot \boldsymbol{c_1} \\ + GBRatio \cdot T^{Gen}\left(GBRatio \cdot \dot{q}_{GeAz}, t\right) + GBRatio \cdot T^{Brake}\left(t\right) \end{bmatrix} \cdot \left[\frac{1}{GBoxEff^{SIGN(LSShftTq)}} - 1\right]$$

Thus,

$$F_r\Big|_{GBFric} = \begin{cases} -T^{GBFric}\left(\ddot{q},\dot{q},q,t\right) & for \quad r = GeAz \\ 0 & otherwise \end{cases}$$

or,

$$F_r\Big|_{GBFric} = \begin{cases} -\begin{bmatrix} GenIner \cdot GBRatio^2 \cdot \ddot{q}_{GeAz} + GenDir \cdot GenIner \cdot GBRatio\, {}^E\boldsymbol{\alpha}^R \cdot \boldsymbol{c_1} \\ + GBRatio \cdot T^{Gen}\left(GBRatio \cdot \dot{q}_{GeAz}, t\right) + GBRatio \cdot T^{Brake}\left(t\right) \end{bmatrix} \cdot \left[\frac{1}{GBoxEff^{SIGN(LSShftTq)}} - 1\right] & for \quad r = GeAz \\ 0 & otherwise \end{cases}$$

or,

$$F_r\Big|_{GBFric} = \begin{cases} -\left(\begin{array}{l} GenIner \cdot GBRatio^2 \cdot \ddot{q}_{GeAz} + GenDir \cdot GenIner \cdot GBRatio \left\{ \left(\sum_{i=4}^{12} {}^E\boldsymbol{\omega}^R_i \ddot{q}_i \right) + \left[\sum_{i=7}^{12} \frac{d}{dt}\left({}^E\boldsymbol{\omega}^R_i \right) \dot{q}_i \right] \right\} \cdot \boldsymbol{c_1} \\ + GBRatio \cdot T^{Gen}\left(GBRatio \cdot \dot{q}_{GeAz}, t\right) + GBRatio \cdot T^{Brake}\left(t\right) \end{array} \right) \cdot \left[\frac{1}{GBoxEff^{SIGN(LSShftTq)}} - 1\right] & for \quad r = GeAz \\ 0 & otherwise \end{cases}$$

Thus,

$$\left.\left[C\left(q,t\right)\right]\right|_{GBFric}=\begin{cases}{}^{E}\boldsymbol{\omega}_{Row}^{G}\cdot\overline{\overline{\boldsymbol{I}}}^{G}\cdot{}^{E}\boldsymbol{\omega}_{Col}^{G}\left[\dfrac{1}{GBoxEff^{SIGN\left(LSShftTq\right)}}-1\right] & for\quad\left(Row=13,Col=4,5,\ldots,13\right)\\ 0 & otherwise\end{cases}$$

$$\left.\left\{-f\left(\dot{q},q,t\right)\right\}\right|_{GBFric}=\begin{Bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ -\left\{{}^{E}\boldsymbol{\omega}_{GeAz}^{G}\cdot\overline{\overline{\boldsymbol{I}}}^{G}\cdot\left[\sum_{i=7}^{12}\dfrac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{i}^{R}\right)\dot{q}_{i}\right]+GBRatio\cdot T^{Gen}\left(GBRatio\cdot\dot{q}_{GeAz},t\right)+GBRatio\cdot T^{Brake}\left(t\right)\right\}\cdot\left[\dfrac{1}{GBoxEff^{SIGN\left(LSShftTq\right)}}-1\right] \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{Bmatrix}$$

It is noted that the gearbox friction generalized active force effects both the mass matrix and the forcing function. Its effect on the mass matrix can be thought of as an apparent mass in the system (i.e., an active friction force which is a function of the generator acceleration). The gearbox friction causes the mass matrix to become unsymmetric. Note that the equation for DOF GeAz is an implicit equation (since the gearbox friction is a function of DOF GeAz), which has no closed-form solution. To avoid the complications resulting from this implicit characterization, the value of the $LSShftTq$ from the previous time step is used in the SIGN() function in place of the value of the $LSShftTq$ in the current time step. This may be done since it may be assumed that $LSShftTq$ will not change much between time steps if the time step is considered small enough. Note that gearbox friction is the only term in the equations of motion that cause the mass matrix to be unsymmetrical. The mass matrix will only be unsymmetric if $GBoxEff$ is not 100%—if $GBoxEff$ is 100%, then the mass matrix is completely symmetric.

Tail-Furl:

The tail-furl springs (linear and stops) and tail-furl dampers (Coulomb, linear, and stops) bring about tail-furl moments.

$$\left. F_r \right|_{SpringTF} = \begin{cases} -TFrlSpr \cdot q_{TFrl} \\ \quad - IF\left[q_{TFrl} > TFrlUSSP, TFrlUSSpr\left(q_{TFrl} - TFrlUSSP \right), 0 \right] & for \quad r = TFrl \\ \quad - IF\left[q_{TFrl} < TFrlDSSP, TFrlDSSpr\left(q_{TFrl} - TFrlDSSP \right), 0 \right] & \\ 0 & otherwise \end{cases}$$

and

$$\left. F_r \right|_{DampTF} = \begin{cases} -TFrlDmp \cdot \dot{q}_{TFrl} - IF\left[\dot{q}_{TFrl} <> 0, TFrlCDmp \cdot SIGN\left(\dot{q}_{TFrl} \right), 0 \right] \\ \quad - IF\left[q_{TFrl} > TFrlUSDP, TFrlUSDmp \cdot \dot{q}_{TFrl}, 0 \right] & for \quad r = TFrl \\ \quad - IF\left[q_{TFrl} < TFrlDSDP, TFrlDSDmp \cdot \dot{q}_{TFrl}, 0 \right] & \\ 0 & otherwise \end{cases}$$

Thus,

$$\left[C\left(q,t \right) \right]\Big|_{SpringTF} = 0$$

$$\left\{ -f\left(\dot{q},q,t \right) \right\}\Big|_{SpringTF} = \left\{ \begin{matrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ -TFrlSpr \cdot q_{TFrl} \\ \quad - IF\left[q_{TFrl} > TFrlUSSP, TFrlUSSpr\left(q_{TFrl} - TFrlUSSP \right), 0 \right] \\ \quad - IF\left[q_{TFrl} < TFrlDSSP, TFrlDSSpr\left(q_{TFrl} - TFrlDSSP \right), 0 \right] \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{matrix} \right\}$$

and

$$\left[C\left(q,t\right)\right]\Big|_{DampTF} = 0$$

$$\left\{-f\left(\dot{q},q,t\right)\right\}\Big|_{DampTF} = \left\{\begin{array}{l} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ -TFrlDmp \cdot \dot{q}_{TFrl} - IF\left[\dot{q}_{TFrl} <> 0, TFrlCDmp \cdot SIGN\left(\dot{q}_{TFrl}\right), 0\right] \\ \quad - IF\left[q_{TFrl} > TFrlUSDP, TFrlUSDmp \cdot \dot{q}_{TFrl}, 0\right] \\ \quad - IF\left[q_{TFrl} < TFrlDSDP, TFrlDSDmp \cdot \dot{q}_{TFrl}, 0\right] \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{array}\right\}$$

Tail:

The rigid lump masses of the tail boom and tail fin bring about generalized inertia forces and generalized active forces associated with the structure weight. Additionally, the tail fin brings about generalized active forces associated with tail fin aerodynamics.

$$F_r^*\Big|_A = {}^E\boldsymbol{v}_r^I\cdot\left(-m^B\,{}^E\boldsymbol{a}^I\right) + {}^E\boldsymbol{\omega}_r^A\cdot\left(-{}^E\dot{\boldsymbol{H}}^A\right) + {}^E\boldsymbol{v}_r^J\cdot\left(-m^F\,{}^E\boldsymbol{a}^J\right)\quad\left(r=1,2,\ldots,22\right)\qquad \text{where}\qquad m^B = BoomMass\qquad\text{and}\qquad m^F = TFinMass$$

Thus,

$$F_r^*\Big|_A = {}^E\boldsymbol{v}_r^I\cdot\left(-m^B\,{}^E\boldsymbol{a}^I\right) + {}^E\boldsymbol{\omega}_r^A\cdot\left(-\overline{\overline{\boldsymbol{I}}}^A\cdot{}^E\boldsymbol{\alpha}^A - {}^E\boldsymbol{\omega}^A\times\overline{\overline{\boldsymbol{I}}}^A\cdot{}^E\boldsymbol{\omega}^A\right) + {}^E\boldsymbol{v}_r^J\cdot\left(-m^F\,{}^E\boldsymbol{a}^J\right)\quad\left(r=1,2,\ldots,22\right)$$

where $\overline{\overline{\boldsymbol{I}}}^A = \left[TFrlIner - m^B\left|\boldsymbol{r}^{WI} - \boldsymbol{r}^{WI}\cdot\boldsymbol{tfa}\,\boldsymbol{tfa}\right|^2\right]\boldsymbol{tfa}\,\boldsymbol{tfa}$ or

$$\overline{\overline{\boldsymbol{I}}}^A = \left(TFrlIner - BoomMass\left\{\begin{array}{l}\left(BoomCMxn - TFrlPntxn\right)^2\left[1-cos^2\left(TFrlSkew\right)cos^2\left(TFrlTilt\right)\right]\\ \quad +\left(BoomCMzn - TFrlPntzn\right)^2 cos^2\left(TFrlTilt\right)\\ \quad +\left(BoomCMyn - TFrlPntyn\right)^2\left[1-sin^2\left(TFrlSkew\right)cos^2\left(TFrlTilt\right)\right]\\ \quad -2\left[\begin{array}{l}\left(BoomCMxn-TFrlPntxn\right)\left(BoomCMzn-TFrlPntzn\right)cos\left(TFrlSkew\right)cos\left(TFrlTilt\right)sin\left(TFrlTilt\right)\\ \quad +\left(BoomCMxn-TFrlPntxn\right)\left(BoomCMyn-TFrlPntyn\right)cos\left(TFrlSkew\right)sin\left(TFrlSkew\right)cos^2\left(TFrlTilt\right)\\ \quad +\left(BoomCMzn-TFrlPntzn\right)\left(BoomCMyn-TFrlPntyn\right)sin\left(TFrlSkew\right)cos\left(TFrlTilt\right)sin\left(TFrlTilt\right)\end{array}\right]\end{array}\right\}\right)\boldsymbol{tfa}\,\boldsymbol{tfa}$$

Or,

$$\begin{aligned}F_r^*\Big|_A = {}&{}^E\boldsymbol{v}_r^I\cdot\left(-m^B\left\{\left(\sum_{i=1}^{11}{}^E\boldsymbol{v}_i^I\ddot{q}_i\right) + {}^E\boldsymbol{v}_{\boldsymbol{TFrl}}^I\ddot{q}_{TFrl} + \left[\sum_{i=4}^{11}\frac{d}{dt}\left({}^E\boldsymbol{v}_i^I\right)\dot{q}_i\right] + \frac{d}{dt}\left({}^E\boldsymbol{v}_{\boldsymbol{TFrl}}^I\right)\dot{q}_{TFrl}\right\}\right)\\ &+{}^E\boldsymbol{\omega}_r^A\cdot\left(-\overline{\overline{\boldsymbol{I}}}^A\cdot\left\{\left(\sum_{i=4}^{11}{}^E\boldsymbol{\omega}_i^A\ddot{q}_i\right) + {}^E\boldsymbol{\omega}_{\boldsymbol{TFrl}}^A\ddot{q}_{TFrl} + \left[\sum_{i=7}^{11}\frac{d}{dt}\left({}^E\boldsymbol{\omega}_i^A\right)\dot{q}_i\right] + \frac{d}{dt}\left({}^E\boldsymbol{\omega}_{\boldsymbol{TFrl}}^A\right)\dot{q}_{TFrl}\right\} - {}^E\boldsymbol{\omega}^A\times\overline{\overline{\boldsymbol{I}}}^A\cdot{}^E\boldsymbol{\omega}^A\right)\quad\left(r=1,2,\ldots,11;TFrl\right)\\ &+{}^E\boldsymbol{v}_r^J\cdot\left(-m^F\left\{\left(\sum_{i=1}^{11}{}^E\boldsymbol{v}_i^J\ddot{q}_i\right) + {}^E\boldsymbol{v}_{\boldsymbol{TFrl}}^J\ddot{q}_{TFrl} + \left[\sum_{i=4}^{11}\frac{d}{dt}\left({}^E\boldsymbol{v}_i^J\right)\dot{q}_i\right] + \frac{d}{dt}\left({}^E\boldsymbol{v}_{\boldsymbol{TFrl}}^J\right)\dot{q}_{TFrl}\right\}\right)\end{aligned}$$

Thus,

$$\left[C(q,t)\right]\Big|_A(Row,Col) = m^B\,{}^E\boldsymbol{v}^{I}_{Row}\cdot{}^E\boldsymbol{v}^{I}_{Col} + {}^E\boldsymbol{\omega}^{A}_{Row}\cdot\overline{\overline{\boldsymbol{I}}}^A\cdot{}^E\boldsymbol{\omega}^{A}_{Col} + m^F\,{}^E\boldsymbol{v}^{J}_{Row}\cdot{}^E\boldsymbol{v}^{J}_{Col}\quad(Row,Col = 1,2,\ldots,11;15)$$

$$\begin{aligned}\left\{-f(\dot{q},q,t)\right\}\Big|_A(Row) = &-m^B\,{}^E\boldsymbol{v}^{I}_{Row}\cdot\left\{\left[\sum_{i=4}^{11}\frac{d}{dt}\left({}^E\boldsymbol{v}^{I}_{i}\right)\dot{q}_i\right]+\frac{d}{dt}\left({}^E\boldsymbol{v}^{I}_{TFrl}\right)\dot{q}_{TFrl}\right\}\\ &-{}^E\boldsymbol{\omega}^{A}_{Row}\cdot\left(\overline{\overline{\boldsymbol{I}}}^A\cdot\left\{\left[\sum_{i=7}^{11}\frac{d}{dt}\left({}^E\boldsymbol{\omega}^{A}_{i}\right)\dot{q}_i\right]+\frac{d}{dt}\left({}^E\boldsymbol{\omega}^{A}_{TFrl}\right)\dot{q}_{TFrl}\right\}+{}^E\boldsymbol{\omega}^{A}\times\overline{\overline{\boldsymbol{I}}}^A\cdot{}^E\boldsymbol{\omega}^{A}\right)\quad(r = 1,2,\ldots,11;15)\\ &-m^F\,{}^E\boldsymbol{v}^{J}_{Row}\cdot\left\{\left[\sum_{i=4}^{11}\frac{d}{dt}\left({}^E\boldsymbol{v}^{J}_{i}\right)\dot{q}_i\right]+\frac{d}{dt}\left({}^E\boldsymbol{v}^{J}_{TFrl}\right)\dot{q}_{TFrl}\right\}\end{aligned}$$

$$F_r\Big|_{GravA} = {}^E\boldsymbol{v}^{I}_{r}\cdot\left(-m^B g\boldsymbol{z}_2\right)+{}^E\boldsymbol{v}^{J}_{r}\cdot\left(-m^F g\boldsymbol{z}_2\right)\quad(r = 3,4,\ldots,11;TFrl)$$

Thus,

$$\left[C(q,t)\right]\Big|_{GravA} = 0$$

$$\left\{-f(\dot{q},q,t)\right\}\Big|_{GravA}(Row) = -m^B g\,{}^E\boldsymbol{v}^{I}_{Row}\cdot\boldsymbol{z}_2 - m^F g\,{}^E\boldsymbol{v}^{J}_{Row}\cdot\boldsymbol{z}_2\quad(Row = 3,4,\ldots,11;15)$$

$$F_r\Big|_{AeroA} = {}^E\boldsymbol{v}^{K}_{r}\cdot\boldsymbol{F}^{K}_{AeroA}+{}^E\boldsymbol{\omega}^{A}_{r}\cdot\boldsymbol{M}^{A}_{AeroA}\quad(r = 1,2,\ldots,11;TFrl)$$

Thus,

$$\left[C(q,t)\right]\Big|_{AeroA} = 0$$

$$\left\{-f(\dot{q},q,t)\right\}\Big|_{AeroA}(Row) = {}^E\boldsymbol{v}^{K}_{Row}\cdot\boldsymbol{F}^{K}_{AeroA}+{}^E\boldsymbol{\omega}^{A}_{Row}\cdot\boldsymbol{M}^{A}_{AeroA}\quad(Row = 1,2,\ldots,11;15)$$

Overall:

Combining the results from the previous sections it is seen that the various portions of the equations of motion are related to the various forces as follows (NOTE: $B=B1+B2$):

$[C(q,t)]=$

$X+HydroX+T+N+R+H+B+A$	$X+HydroX+T+N+R+H+B+A$	$X+HydroX+T+N+R+H+B+A$	$X+HydroX+T+N+R+H+B+A$	$X+HydroX+T+N+R+H+B+A$	$X+HydroX+T+N+R+H+B+A$	$T+N+R+H+B+A$	$T+N+R+H+B+A$	$T+N+R+H+B+A$	$T+N+R+H+B+A$	$N+R+H+B+A$	$R+H+B$	$H+B$	$H+B$	A	$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
	$X+HydroX+T+N+R+H+B+A$	$X+HydroX+T+N+R+H+B+A$	$X+HydroX+T+N+R+H+B+A$	$X+HydroX+T+N+R+H+B+A$	$X+HydroX+T+N+R+H+B+A$	$T+N+R+H+B+A$	$T+N+R+H+B+A$	$T+N+R+H+B+A$	$T+N+R+H+B+A$	$N+R+H+B+A$	$R+H+B$	$H+B$	$H+B$	A	$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
		$X+HydroX+T+N+R+H+B+A$	$X+HydroX+T+N+R+H+B+A$	$X+HydroX+T+N+R+H+B+A$	$X+HydroX+T+N+R+H+B+A$	$T+N+R+H+B+A$	$T+N+R+H+B+A$	$T+N+R+H+B+A$	$T+N+R+H+B+A$	$N+R+H+B+A$	$R+H+B$	$H+B$	$H+B$	A	$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
			$X+HydroX+T+N+R+H+B+G+A$	$X+HydroX+T+N+R+H+B+G+A$	$X+HydroX+T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$N+R+H+B+G+A$	$R+H+B+G$	$H+B+G$	$H+B$	A	$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
				$X+HydroX+T+N+R+H+B+G+A$	$X+HydroX+T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$N+R+H+B+G+A$	$R+H+B+G$	$H+B+G$	$H+B$	A	$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
					$X+HydroX+T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$N+R+H+B+G+A$	$R+H+B+G$	$H+B+G$	$H+B$	A	$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
						$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$N+R+H+B+G+A$	$R+H+B+G$	$H+B+G$	$H+B$	A	$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
							$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$N+R+H+B+G+A$	$R+H+B+G$	$H+B+G$	$H+B$	A	$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
								$T+N+R+H+B+G+A$	$T+N+R+H+B+G+A$	$N+R+H+B+G+A$	$R+H+B+G$	$H+B+G$	$H+B$	A	$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
									$T+N+R+H+B+G+A$	$N+R+H+B+G+A$	$R+H+B+G$	$H+B+G$	$H+B$	A	$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
										$N+R+H+B+G+A$	$R+H+B+G$	$H+B+G$	$H+B$	A	$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
											$R+H+B+G$	$H+B+G$	$H+B$		$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
			$+GBFric$	$+GBFric$	$+GBFric$	$+GBFric$	$+GBFric$	$+GBFric$	$+GBFric$	$+GBFric$	$+GBFric$	$H+B+G+GBFric$	$H+B$		$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
													$H+B$		$B1$	$B1$	$B1$	$B2$	$B2$	$B2$	$H+B$
														A							
															$B1$	$B1$	$B1$				$B1$
																$B1$	$B1$				$B1$
																	$B1$				$B1$
																		$B2$	$B2$	$B2$	$B2$
																			$B2$	$B2$	$B2$
																				$B2$	$B2$
SYM (except	for GBFric)																				$H+B$

$$\left\{-f\left(\dot{q},q,t\right)\right\}=\left\{\begin{array}{l}
X+HydroX+T+AeroT+N+R+H+B+AeroB+A+AeroA \\
X+HydroX+T+AeroT+N+R+H+B+AeroB+A+AeroA \\
X+GravX+HydroX+T+GravT+AeroT+N+GravN+R+GravR+H+GravH+B+GravB+AeroB+A+GravA+AeroA \\
X+GravX+HydroX+T+GravT+AeroT+N+GravN+R+GravR+H+GravH+B+GravB+AeroB+G+A+GravA+AeroA \\
X+GravX+HydroX+T+GravT+AeroT+N+GravN+R+GravR+H+GravH+B+GravB+AeroB+G+A+GravA+AeroA \\
X+GravX+HydroX+T+GravT+AeroT+N+GravN+R+GravR+H+GravH+B+GravB+AeroB+G+A+GravA+AeroA \\
T+ElasticT+DampT+GravT+AeroT+N+GravN+R+GravR+H+GravH+B+GravB+AeroB+G+A+GravA+AeroA \\
T+ElasticT+DampT+GravT+AeroT+N+GravN+R+GravR+H+GravH+B+GravB+AeroB+G+A+GravA+AeroA \\
T+ElasticT+DampT+GravT+AeroT+N+GravN+R+GravR+H+GravH+B+GravB+AeroB+G+A+GravA+AeroA \\
T+ElasticT+DampT+GravT+AeroT+N+GravN+R+GravR+H+GravH+B+GravB+AeroB+G+A+GravA+AeroA \\
SpringYaw+DampYaw+N+GravN+R+GravR+H+GravH+B+GravB+AeroB+G+A+GravA+AeroA \\
SpringRF+DampRF+R+GravR+H+GravH+B+GravB+AeroB+G \\
H+GravH+B+GravB+AeroB+Gen+Brake+G+GBFric \\
H+GravH+B+GravB+AeroB+ElasticDrive+DampDrive \\
SpringTF+DampTF+A+GravA+AeroA \\
B1+ElasticB1+DampB1+GravB1+AeroB1 \\
B1+ElasticB1+DampB1+GravB1+AeroB1 \\
B1+ElasticB1+DampB1+GravB1+AeroB1 \\
B2+ElasticB2+DampB2+GravB2+AeroB2 \\
B2+ElasticB2+DampB2+GravB2+AeroB2 \\
B2+ElasticB2+DampB2+GravB2+AeroB2 \\
SpringTeet+DampTeet+H+GravH+B+GravB+AeroB
\end{array}\right\}$$